**Seizo Motora.**

## PROPOSED MANŒUVRABILITY INDICES AS A MEASURE OF THE STEERING QUALITIES OF SHIPS

*Introduction.*

As a factor to indicate the manoeuvrability of ships, the radius or diameter of the turning circle, tactical diameter, advance, transfer, etc. have been used. On the other hand, a proposal has been made by Kempf to adopt zigzag test (serpentine test) to evaluate the manoeuvrability of ships in a comprehensive sense.

The present situation is accordingly such that there have been proposed quite a number of qualities of different kinds and characteristics, and their correlation with each other has not yet been clarified. If, therefore, an index of a character which is common to all these qualities and is considered to represent them is obtained, it will be needless to mention here its usefulness.

Nomoto [2], [3], etc., commenced their study on the application of frequency response approach to the steering motion of ships, and then attempted to express the manœuvrability of ships as a whole in terms of two indices—the one indicating the turning ability and the other indicating the course stability or quick responsibility.

Using these two indices, they succeeded to clarify various characteristics of manoeuvrability of ships had been uncertain so far. This subject has been further studied and developed by Nomoto [5], [6], Takahashi, Kawashima [4] Motora [7], etc.

One of the important subjects to be discussed at the present subcommittee meetings of the 9th I.T.T.C. is the determination of the manoeuvrability indices of ships. Under these circumstances, the author would like to introduce the outline of investigations undertaken and developed in Japan on the manoeuvrability of ships, which, he proposes, could be taken up at the meeting for discussion.

1. *The possibility of developing a first order approximation on the steering motion.*

When the Davidson's notations [1] are used, the equation of motion is expressed by:

$$\left.\begin{aligned} \left(\frac{l}{V}\right) m_2 \frac{d\psi}{dt} + Ce\,\psi - \left(\frac{l}{V}\right) m\,\dot{\theta} &= C\lambda \cdot \delta \\ \left(\frac{l}{V}\right)^2 n \frac{d\dot{\theta}}{dt} + \left(\frac{l}{V}\right) C_k \dot{\theta} - C_m \psi &= C\mu \cdot \delta \end{aligned}\right\} \quad (1)$$

Where: $\dot{\theta}$ = Angular velocity of turning motion.
$\delta$ = Helm angle.
$\psi$ = Drift angle.
$V$ = Ship's speed.
$l$ = Length of ship.

Eliminating $\psi$ from equation (1), then:

$$T_1 T_2 \frac{d^2\dot{\theta}}{dt^2} + (T_1 + T_2)\frac{d\dot{\theta}}{dt} + \dot{\theta} = K\delta + KT_3 \frac{d\delta}{dt} \quad (2)$$

Where :

$$\left.\begin{aligned} K &= \left(\frac{V}{l}\right) \frac{C_m C_\lambda + C_l C_\mu}{C_l C_k - m C_m} \\ T_1 \cdot T_2 &= \left(\frac{l}{V}\right)^2 \frac{m_2\ n}{C_l C_k - m C_m} \\ T_1 + T_2 &= \left(\frac{l}{V}\right) \frac{m_2 C_k + n C_l}{C_l C_k - m C_m} \\ T_3 &= \left(\frac{l}{V}\right) \frac{m_2 C_\mu}{C_m C_\lambda + C_l C_\mu} \end{aligned}\right\} \quad (2')$$

$T_1$ and $T_2$ mentioned above have close connection with the course stability of ships, and have the following relation with the Davidson's course stability index.

$$\left.\begin{aligned} T_1 &= -\frac{1}{p_1}\left(\frac{l}{V}\right) \\ T_2 &= -\frac{l}{p_2}\left(\frac{l}{V}\right) \end{aligned}\right\} \quad (3)$$

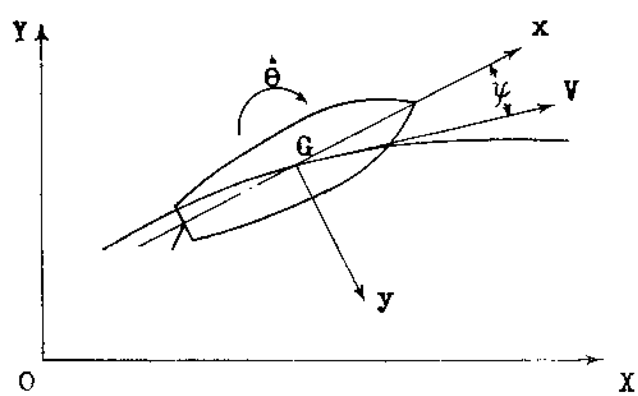



FIG. 1.

Nomoto, etc. [3] have disclosed that, of the coefficients $T_1$, $T_2$, $T_3$ and K, $T_1$ and K are predominant factors contributing to the manoeuvrability, and that equation (2) can be expressed fundamentally by the following differential equation of the first order regarding $\theta$.

$$T \frac{d\theta}{dt} + \theta = K \cdot \delta \qquad (4)$$

This theory is based upon the following analysis. When obtained from the exact solution of equation (2) is compared with the solution of equation (4), the difference between these solutions becomes zero after sufficient lapse of time, if:

$$T = T_1 + T_2 - T_3 \qquad (5)$$

They have disclosed that the value of T is very close to $T_1$ in any ship, ($T = 0.8 \;\; 0.9\, T_1$) and that the difference between equation (2) and (4) is not so great even when the lapse of times is not sufficient.

The comparison between the solutions of equations (2) and (4) as two examples is as shown in Fig. 2, which indicates that it would be possible to develop a first order approximation on the steering motion.

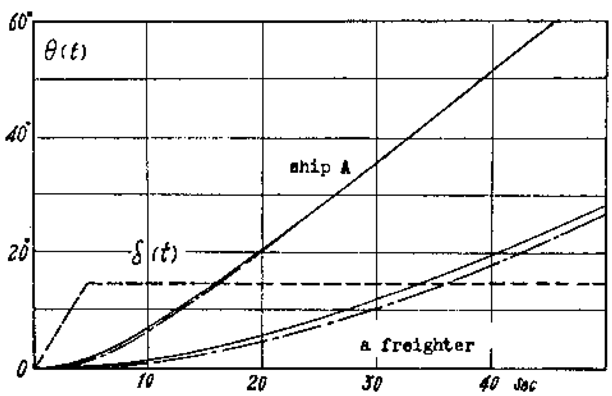



FIG. 2.

If the fundamental motion due to steering can be approximated by a first order system as shown in Fig. 2, any steering motion must likewise be approximated by equation (4) except at very early stage, inasmuch as any motion is the superposition of these fundamental motions. Nomoto paid attention to this fact, and attempted to develop a first approximation from the results of Kempf's zigzag test (standard test).

Fig. 3 shows the values of T and K so determined as to give the closest approximation, which have been obtained from the analysis of the results of zigzag test on a cargo ship in trial condition (Fig. 3-*a*) and a tanker in full load condition (Fig. 3-*b*).

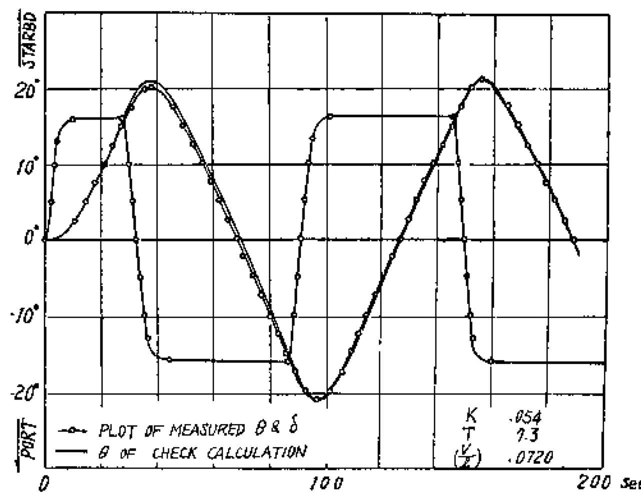



FIG. 3 *a*.

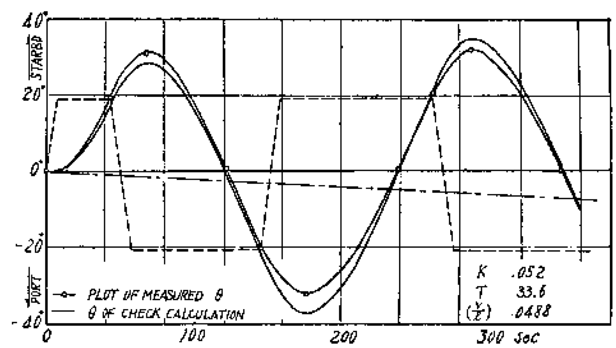



FIG. 3 *b*.

Standard Manoeuvre Test for a full-loaded Tanker.

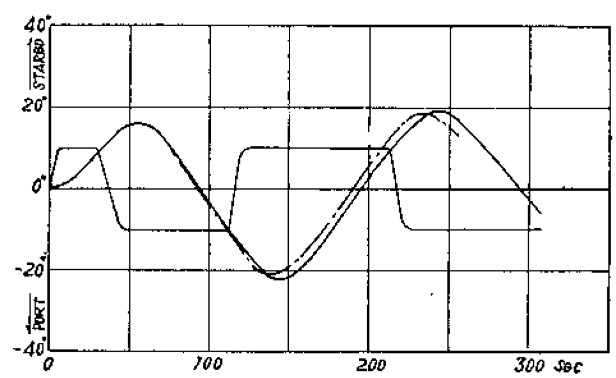



FIG. 3 *c*.

Standard Manoeuvre Test for a full-loaded Cargo Boat.

It may be noted from Fig. 3 that in case of a cargo ship, which is comparatively directionally stable, almost perfect approximation can be made, and further that in case of a tanker, which is rather directionnally less stable, an approximation, though not so close as in case of a cargo ship but sufficiently applicable for practical purpose, can also be made.

On the other hand, an attempt has been made by Motora [7] to explain, from a different standpoint, that the steering motion can be fundamentally approximated by an equation of motion in a first order system. It has been well known that the motion of ships can be resolved into a rotating motion about an axis of instantaneous rotation of ships, which is so called the pivoting point, and a progressive motion along the longitudinal axis of the ship. The ship appears to move along the tangent to the turning path at the pivoting point. When the rudder is suddenly put over, this pivoting point coincides with the centre of percussion when the rudder is subjected to an impact normal to the rudder plane [8]. Then the centre of pressure gradually moves forward as the ship turns, but the shift of this point is not so big. Accordingly the drift angle $\psi$ in equation (1) does not take an arbitrary value, but changes with the change of $\theta$, keeping the relation of:

$$l_p\,\dot{\theta} = V\psi \qquad (6)$$

Where : $l_p$ = Distance from the centre of gravity of ship to the pivoting point.

Substituting equation (6) for equation (1) (the second equation), we obtain:

$$\left(\frac{l}{V}\right)^2 n\,\frac{d\dot{\theta}}{dt} + \left(\frac{l}{V}\right)\left(C_k - \frac{l_p}{l}C_m\right)\dot{\theta} = C\mu \cdot \delta \qquad (7)$$

$$\left(\frac{l}{V}\right)\frac{n}{C_k - \frac{l_p}{l}C_m}\,\frac{d\dot{\theta}}{dt} + \dot{\theta} = \left(\frac{l}{V}\right)\frac{C\mu}{C_k - \frac{l_p}{l}C_m}\cdot\delta \qquad (8)$$

In equation (8), put

$$\left.\begin{aligned} \left(\frac{l}{V}\right)\frac{n}{C_k - \frac{C_b}{l}C_m} &= T \\ \left(\frac{V}{l}\right)\frac{C\mu}{C_k - \frac{l_p}{l}C_m} &= K \end{aligned}\right\} \qquad (9)$$

then equation (8) coincides with equation (4), and the following relation exists between T and K.

$$\frac{T}{K} = \frac{C\mu}{n}\left(\frac{V}{l}\right)^2 \qquad (10)$$

Since in the actual steering motion the pivoting point moves forward with the increase of the angular velocity of turning motion, T and K in equation (9) gradually change keeping the relation as given by equation (10). Accordingly, equation (8) is not exactly equivalent to equation (4), but may serve to understand that the steering motion can be fundamentally approximated by a differential equation of the first order.

2. *Manoeuvrability indices.*

As mentioned in § 1, the steering motion of ships can be approximated by a first order system, such as equation (4). This equation very much resembles the equation for the transient current when a circuit is closed.

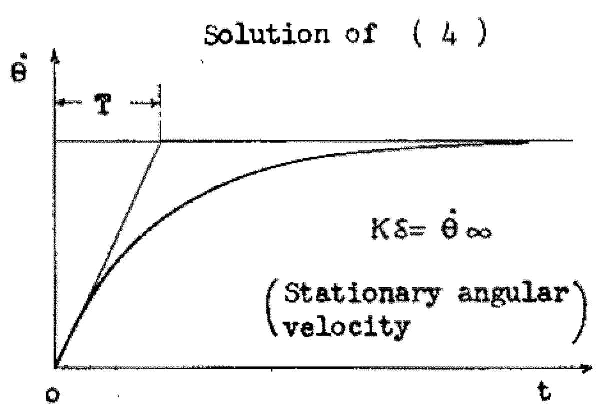



FIG. 4*a*.

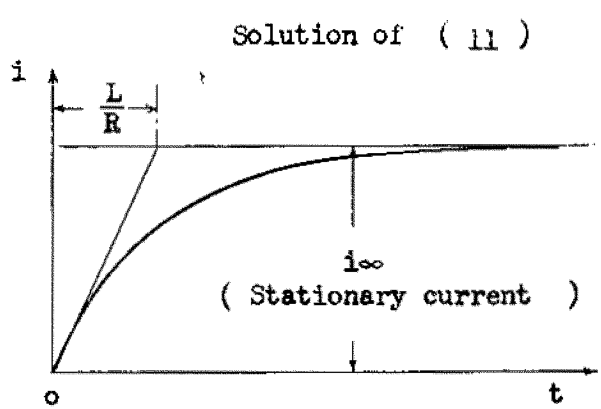



FIG. 4*b*.

When a circuit, having inductance L, electromotive power E and resistance R, is closed, the electric current $i$ is given by:

$$\frac{L}{R}\frac{di}{dt} + i = \frac{E}{R} \qquad (11)$$

When this equation is compared with equation (4), it may be noted that L corresponds to L/R and K to E/R, Accordingly it is considered that *K indicates the angular velocity eventually reached, and therefore the turning ability,* and *T indicates the quickness to move from one stationary state of another stationary state, i. e. quick responsibility,* since T corresponds to the time lag till the ship reaches the stationary state, i.e. time constant.

On the other hand, T is nearly equal to the index of course stability as shown in equation (5), so that *T is also considered to be an index indicating the course stability of ships.*

The comparison between the solutions of equation (4) and (11) is as shown in Figure 4, which may sufficiently explain the aforementioned relationship.

Figure 5 indicates the angle θ obtained by integrating the curve in Figure 4-*a*, where the helm angles have been taken stepwise for the sake of simplicity.

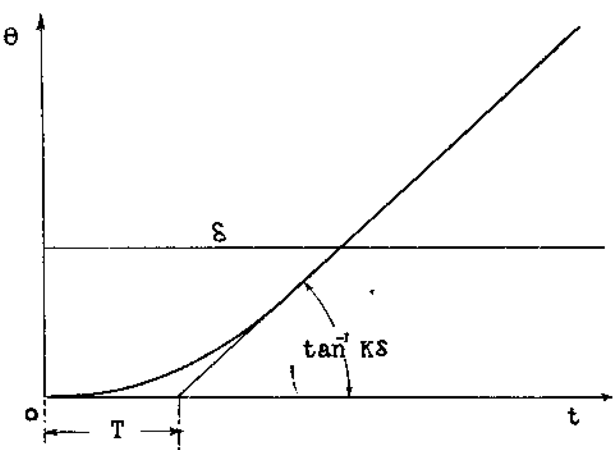



FIG. 5.

When the rudder is suddenly put to the mid-position, the angular velocity changes in a similar manner as the electric current decreases when the electro motive force is made zero. The angular displacement from the direction of the ship's course when the rudder is put to the mid-position to the direction that the ship eventually proceeds becomes $K\delta T$. (See Fig. 6).

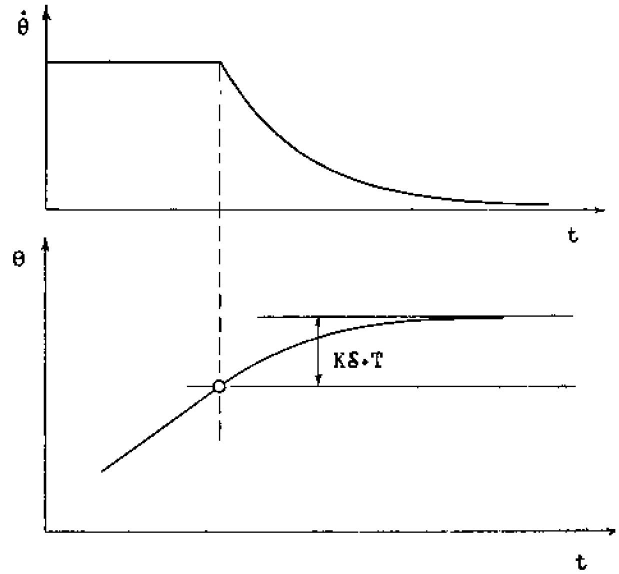



FIG. 6.

The angular displacement from the direction of the ship's course when the rudder is put over to the direction when the angular velocity becomes zero (over swinging angle) obtained from the zigzag test, which has been proposed by Kempf as a measure of manoeuvrability, corresponds to $K\delta T/2$. Accordingly the manoeuvrability of ships as a whole can not be sufficiently represented by this over swinging angle only, but it is necessary to couple this angle with another index, for instance, K which represents the stationary turning.

In summary, the manoeuvrability of ships can be indicated almost perfectly by two indices, T and K.

T: represents Quick responsibility or Course stability.

| | | | |
|---|---|---|---|
| T | ↑ greater | Quick responsibility and Course stability | ↑ worse |
| | ↓ smaller | | ↓ better |

K: represents Turning ability, having the relation $K\delta = \dot{\theta}\infty$ (stationary).

| | | | |
|---|---|---|---|
| K | ↑ greater | Turning ability | ↑ better |
| | ↓ smaller | | ↓ worse |

3. *Relation of T and K with the turning path.*

To show the relation of T and K with the turning path, the locus of the C. G. of a ship in turning for various combinations of T and K are computed. Ship

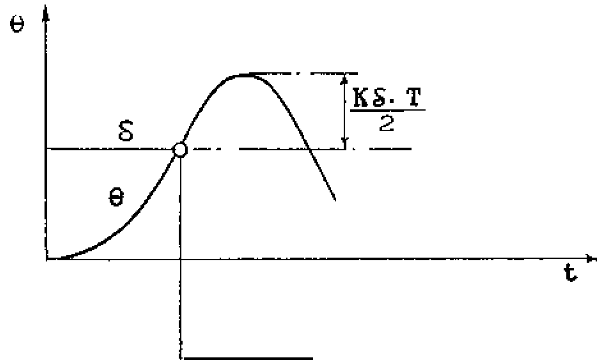



FIG. 7.

speed in that computation is assumed to be 10 m/sec and the rudder is assumed to be put over to 35° suddenly when the ship is running straight. (fig. 8, 9 and 10.)

Advance, transfer, tactical diameter, etc. must be expressed by a function of T and K with regard to the expression of this function in a simple form, investigation are being made by Nomoto and Motora.

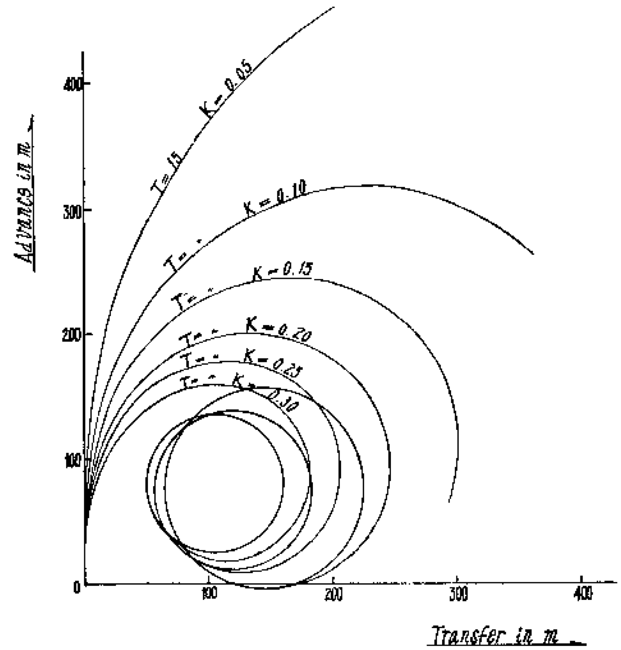



FIG. 9.

T is constant, K is varied.

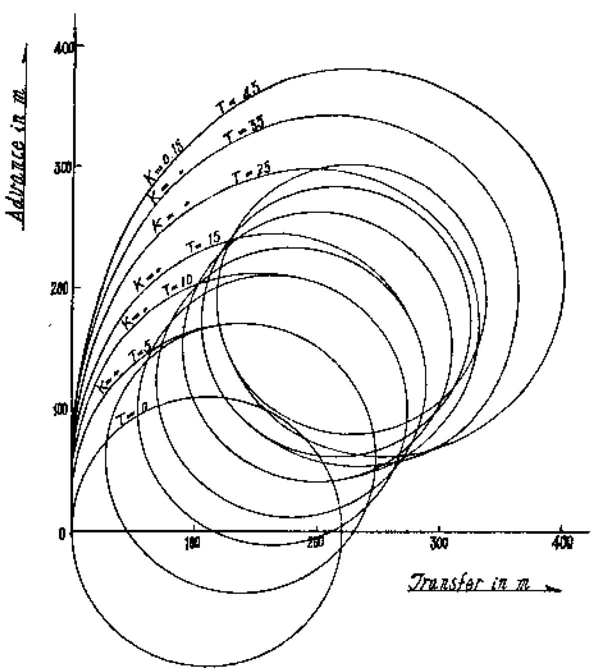



FIG. 8.

K is constant, T is varied.

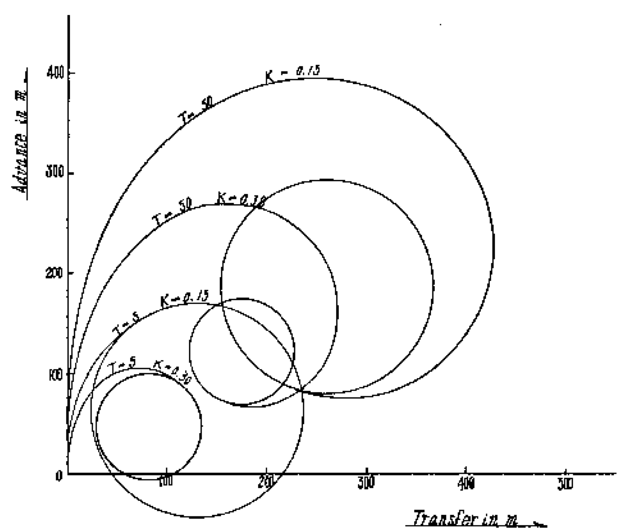



FIG. 10.

K and T are varied.

4. *A method to obtain T and K by experiments and their actual values.*

1. Method to obtain T and K by zigzag test.

The indices T and K can be calculated from equation (2′) and (5) using the fundamental resistance derivatives of ships, but the advantage of these indices is rather such that they can easily be derived from the results of manoeuvre test with actual or model ships.

Nomoto developed a method to obtain T and K as a result of analysing the Kempf's zigzag test, which is published in his paper [3], but this method was rather complicated.

Then he succeeded to simplify the above method, and proposed a simple and very practical method [5] which is considered at present to be the most appropriate method to obtain these indices. In the following is described the outline of this method.

The equation of motion is rewritten as:

$$T \frac{d\dot{\theta}}{dt} + \dot{\theta} = K\delta + K\delta_r \qquad (12)$$

where $\delta r$ is the error of the helm angle, which is substituted for the effect of initial velocity, slight unbalanced moment, etc. This term has been defined by Nomoto as residual helm.

Integrating equation (12), then:

$$T\dot{\theta} + \theta = K \int_0^t \delta\, dt + K\delta_r \cdot t \qquad (13)$$

On the other hand, the time-turning angle diagram is recorded, where the angle and time are denoted as shown in Fig. 11.

*i)* $\theta = \theta_e$ and $\dot{\theta} = 0$ at $t = t_e$, then equation (13) becomes:

$$\theta_e = K \int_0^{te} \delta\, dt + K\delta t \cdot t_e \qquad (14)$$

$\theta = \theta'_e$ and $\dot{\theta}' = 0$ at $t = t'_e$, then

$$\theta'_e = K \int_0^{t'_e} \delta\, dt + K\delta t \cdot t'_e \qquad (15)$$

From equations (14) and (15), K and $\delta r$ can be obtained.

*ii)* $\theta = 0$ and $\dot{\theta} = \dot{\theta}_o$ (obtained from the slope of the curve recorded at $t = t_o$), then:

$$T\dot{\theta} = K \int_0^{te} \delta\, dt + K\delta t \cdot t_o \qquad (16)$$

T can be calculated from equation (16), and thus all of K , T and $\delta r$ can be obtained.

In the actual calculation, an arrangement has been made in such a way that the foregoing process is made with the former-half and latter-half periods of the test separately, and two sets of K, T and $\delta r$ thus obtained are averaged.

T has the dimension of time and K of 1/time, so that it may be more convenient, for comparison purpose, to express these values in dimensionless forms, namely:

$$\left.\begin{aligned} T' &= T \times \frac{V}{l} \\ K' &= K \times \frac{l}{V} \end{aligned}\right\} \qquad (17)$$

The values of T and K obtained by Nomoto according to the above method as to many ships are as shown in Table 1 (from ref. [9]).

2) Method to obtain T and K by turning test.

This method is to obtain T and K using the relations as given in Figure 5 and Figure 6. This is a direct and appropriate method to obtain T, but the previous method by the zigzag test may be more suitable for obtaining T, since it is an index indicating the transient state.

The method of test is to put over the rudder as quickly as possible when the ship is advancing on a straight course, and then hold the rudder at a fixed angle. After the turning of the ship becomes to the stationary state, the rudder is put to the mid-position as quickly as possible.

From the record of the turning angle $\theta$ and the time during the above test, T and K can be obtained as shown in Figure 12.

The problem of this method is that, due to the drop of ship's speed, the $t - \theta$ curve does not become to a straight line at large values of $t$, but becomes to a curve convex to the upper side, which causes the error of the measured value of T.

To overcome the above shortcoming, a modification as shown in Figure 13 has been proposed by Motora.

The solid line in Figure 13 indicates the angular displacement of the direction of ship's motion actually measured. On the other hand, the progressive speed of the ship gradually decreases as shown in Figure 13-*b*. When the time base is changed from $dt$ to $dt_1 = \frac{V}{V_0} dt$, then it can be easily proved that the ship's motion corresponds to the speed $V_0$. Accordingly when the

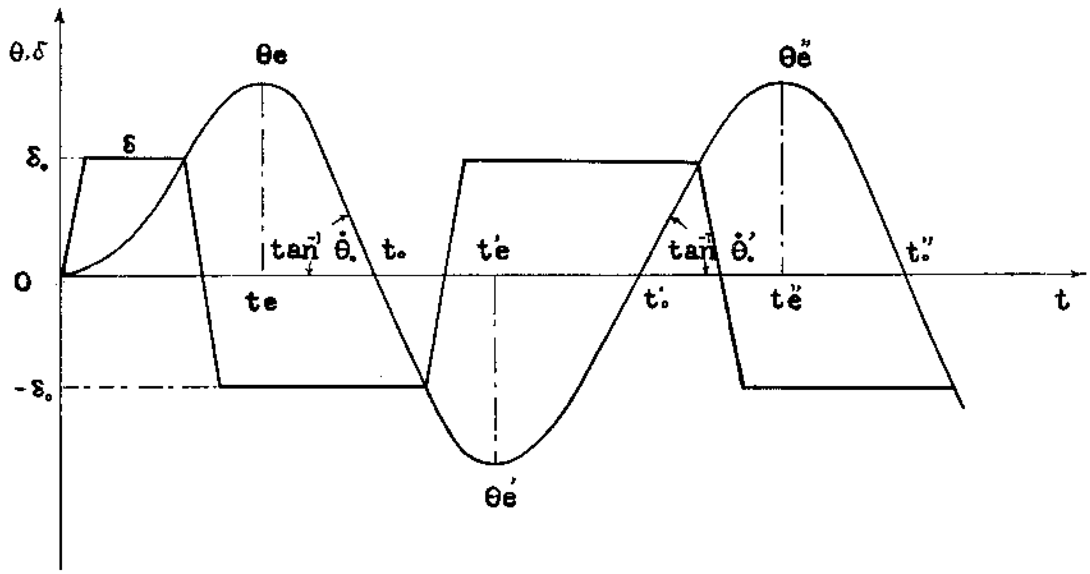

θ,δ
θe
θe″
δ
δ₀
tan⁻¹ θ̇₀
t₀
t′e
tan⁻¹ θ̇₀′
t₀″
O
te
t₀′
te″
t
-δ₀
θe′

FIG. 11.

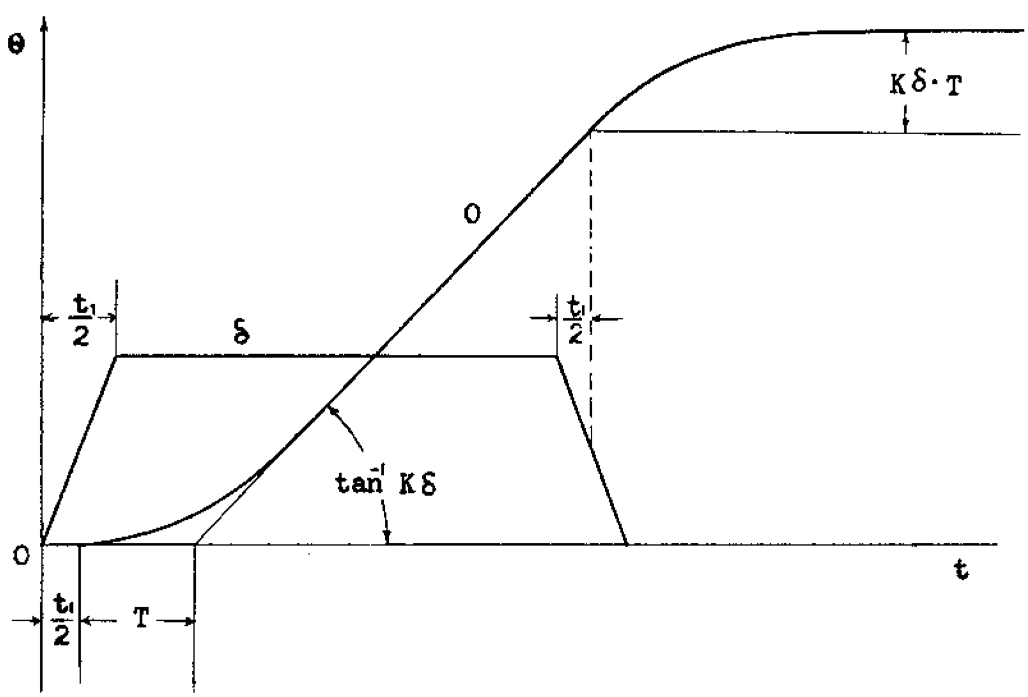

θ
Kδ·T
O
t₁/2
δ
t₁/2
tan⁻¹ Kδ
O
t
t₁/2
T

FIG. 12.

TABLE. 1 — Standard manoeuvre test results

| KIND OF SHIP | COND. | L × B × D | $d_a$ | $d_f$ | $d$ | $\Delta$ | $V_{kt}$ | $\nabla/L^2d$ | $_R/Ld$ | $\delta_0$ | K | T | K′ | T′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | B | 152×20.6×12.7 | 5.61 | 2.44 | 4.02 | 8,828 | 17.2 | .093 | 1/36.3 | 15 | .043 | 11.0 | .74 | .64 |
| | | | | | | | | | | 20 | .041 | 10.0 | .70 | .58 |
| C | B | 114×16.4× 9.3 | 4.58 | 1.96 | 3.27 | 4,180 | 15.7 | .096 | 1/30.2 | 15 | .054 | 6.9 | .76 | .49 |
| C | F | 157×19.6×12.5 | 8.60 | 7.90 | 8.25 | 16,000 | 17.1 | .077 | 1/69.6 | 10 | .071 | 26.2 | 1.27 | 1.47 |
| | | | | | | | | | | —10 | .073 | 26.7 | 1.30 | 1.50 |
| C | H | 121×16.2 | 6.36 | 3.94 | 5.15 | 6,750 | 10.0 | .087 | | 15 | .040 | 18.0 | .94 | .76 |
| | | | | | | | | | | —15 | .035 | 17.0 | .82 | .72 |
| C | F | 133×18.6×10.4 | 8.62 | 7.58 | 8.10 | 15,160 | 14.3 | .103 | 1/65.2 | 10 | .102 | 53.2 | 1.84 | 2.94 |
| | | | | | | | | | | —10 | .085 | 46.9 | 1.54 | 2.59 |
| C | F | 140×19.0×10.5 | 8.70 | 8.00 | 8.35 | 16,050 | 15.0 | .096 | 1/59.9 | 10 | .094 | 35.0 | 1.70 | 1.93 |
| C | H | 94×13.7× 7.6 | 5.26 | 3.04 | 4.15 | 3,800 | 10.3 | .101 | 1/34.2 | 10 | .059 | 20.0 | 1.04 | 1.18 |
| | | | | | | | | | | —10 | .059 | 19.0 | 1.04 | 1.08 |
| C | F | 112×16.2× 9.0 | 7.84 | 7.00 | 7.42 | 9,980 | 11.5 | .105 | 1/64.2 | 10 | .119 | 47.9 | 2.25 | 2.53 |
| T | F | 185×25.2×13.4 | 10.50 | 10.10 | 10.30 | 37,695 | 15.5 | .104 | 1/75.4 | 10 | .086 | 95.0 | 2.00 | 4.09 |
| | | | | | | | | | | —10 | .063 | 42.4 | 1.46 | 1.83 |
| O. T | B | 216×30.6×15.4 | 7.47 | 3.07 | 5.27 | 26,900 | 18.0 | .107 | 1/37.5 | 15 | .027 | 11.4 | .63 | .49 |
| | | | 10.29 | 10.21 | 10.25 | 56,250 | 17,6 | .115 | 1/71.1 | 20 | .056 | 61.3 | 1.33 | 2.57 |
| | F | | 10.31 | 10.32 | 10.32 | 56,700 | 17,6 | .115 | 1/71.1 | 10 | .171 | 21.5 | 4.08 | 9.01 |
| O. T | F | 216×30.6×15.4 | 10.44 | 10.43 | 10.44 | 57,100 | 17.2 | .115 | 1/69.4 | 20 | .050 | 68.7 | 1.22 | 2.82 |
| | | | | | | | | | | 35 | .040 | 58.1 | .97 | 2.38 |
| T | F | 192×26.5×13.87 | 10.43 | 10.43 | 10.43 | 43,100 | 16.0 | .109 | 1/72.1 | 10 | .073 | 80.5 | 1.70 | 3.44 |
| | | | | | | | | | | 15 | .060 | 56.5 | 1.39 | 2.44 |
| | | | 10.61 | 10.60 | 10.60 | 44,090 | 16.1 | .110 | | —15 | .051 | 47.1 | 1.17 | 2.03 |
| T | 2/3 F | 106×16.2× 8.0 | 5.45 | 5.10 | 5.27 | 6,928 | 14.0 | .114 | 1/51.5 | 10 | .110 | 41.6 | 1.62 | 2.83 |
| | | | | | | | | | | —10 | .103 | 34.6 | 1.52 | 2.35 |
| T | F | 154×20.0×11.5 | 8.85 | 9.18 | 9.02 | 20,583 | 12.3 | .094 | 1/71.8 | 10 | .094 | 72.5 | 2.31 | 2.96 |
| | | | | | | | | | | —10 | .091 | 72.4 | 2.22 | 2.96 |
| T | F | 192×26.8×13.7 | 10.60 | 10.20 | 10.40 | 43,000 | 15.0 | .109 | 1/74.2 | 10 | .063 | 83.8 | 1.58 | 3.36 |
| | | | | | | | | | | 10 | .037 | 9.85 | .87 | .42 |
| | B | | 6.44 | 1.85 | 4.14 | 13,000 | 16.0 | .083 | 1/29.6 | —10 | .044 | 10.6 | 1.02 | .45 |
| W | Arival | 57× 9.7× 5.1 | 5.12 | 3.62 | 4.37 | 1,304 | | .090 | 1/27.7 | —15 | .199 | 5.2 | 1.29 | .80 |
| W | Trial | 57× 9.7× 5.1 | 4.56 | 2.34 | 3.45 | 935 | 17.4 | .081 | 1/21.8 | —15 | .155 | 4.1 | .99 | .64 |
| Emigrant ship | 1/2 F | 145×20.4×11.9 | 7.20 | 5.50 | 6.35 | 12,070 | 19.0 | .018 | 1/49.7 | 15 | .053 | 11.8 | .78 | .80 |
| C | F | 75×11.9× 5.5 | 4.80 | 4.78 | 4.79 | 3,222 | 10.0 | .118 | 1/55.2 | 10 | .077 | 25.4 | 1.10 | 1.75 |
| | | | | | | | | | | —10 | .093 | 22.9 | 1.40 | 1.58 |
| Train Ferry | H | 111×17.4× 6.8 | 4.92 | 4.64 | 4.78 | 5,370 | 14.4 | .089 | 1/30.1 | 15 | .096 | 22.6 | 1.44 | 1.51 |
| Coastguard Cutter | H | 51.5× 7.7× 4.5 | 2.89 | 2.58 | 2.73 | 534 | 13.0 | .072 | 1/40.0 | 10 | .206 | 13.1 | 1.66 | 1.62 |
| | | | | | | | | | | 20 | .162 | 12.2 | 1.42 | 1.39 |
| | | | | | | | | | | 30 | .155 | 13.8 | 1.55 | 1.38 |

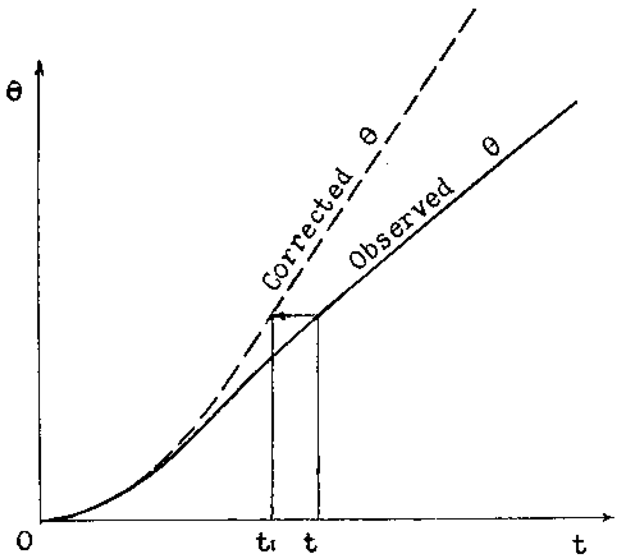



FIG. 13*a*.

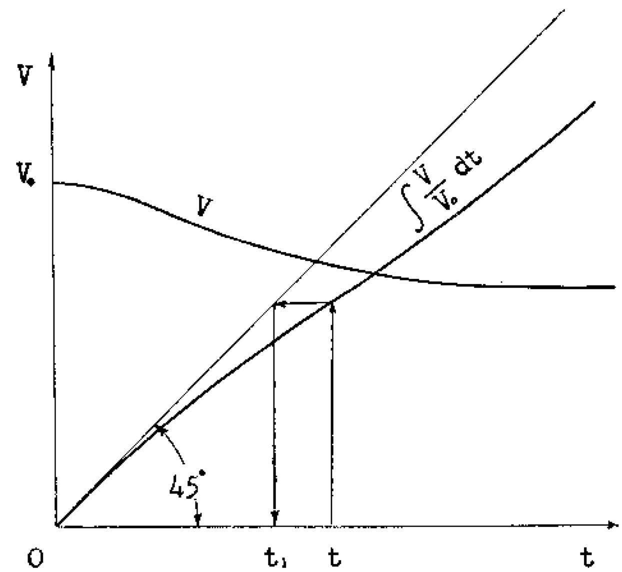



FIG. 13*b*.

time base is changed from $t$ to $t_1 = \int \frac{V}{V_0} dt$ by integrating $\frac{V}{V_0}$ d$t$, or, in other words, when $\int \frac{V}{V_0}$ d$t$ curve is drawn as shown in Figure 13*b*, then $t$ can easily be reduced to $t_1$. If, therefore, the measured value of θ in Figure 13*a* corresponding to the time $t$ is shifted to the value corresponding to $t_1$, θ-time curve corresponding to the constant speed V can be drawn.

This modification is applicable not only to turning motion, but also to any steering motion.

5. *Relation between T and K (Relation between the course stability and the turning ability).*

While several kinds of turning path under many combinations of T and K are shown in § 3, in actual case, it will be noticed from experimental values of T & K shown in Table 1 that there is certain narrow range for the combination of T and K values.

Figure 14*a* shows the relation between 1/T′, and 1/K′, the former represents the course stability and the latter represents the turning ability of ships.

In Figure 14*a* plotted values are scattered within a range limited by two radial lines through origin.

Though the range bounded by two radial lines in Figure 14*a* is not narrow, it will be easily found that ships plotted on the upper part of Figure 14*a* have smaller rudder area ratios AR/L. d, and ships plotted on lower part have larger rudder area ratios. This fact indicates that if we choose $1/K', \frac{AR}{ld}$ for the ordinate instead of 1/K′, the scattered plots will gather around a radial line through origin.

Figure 14*b* shows the result.

Addingly, Nomoto [9] modified the ordinate as :

$$\frac{1}{K'}\frac{A_R}{L \cdot d} \times \frac{l^2 d}{\nabla} = \frac{A_R \cdot l}{K' \nabla} \qquad (17)$$

Where $\nabla$ = volume of displacement.
thinking $l^2d/\nabla$ as an important factor wich will give an effect upon K′ value. The result is shown in Figure 14(c), in which concentration of plots into a radial line is acknowledged.

Therefore, relation between T′ and K′ may be expressed in the following formula;

$$K = \begin{pmatrix} \text{almost invariable constant} \\ \text{for all kinds of usual ships} \end{pmatrix} \times \frac{A_R}{l \cdot d}\frac{l^2 d}{\nabla} \times T' \qquad (18)$$

On the other hand, the above relation between K and T can also be presumed from equation (10), namely :

$$\frac{K}{T} = \frac{C\mu}{n}\left(\frac{V}{l}\right)^2 \qquad (10)$$

This equation may not represent the ratio K/T having the significance as given in Figure 5, when the shift of pivoting point according to the lapse of time for turning motion is taken into account. It is, however, considered that this difference can be substan-

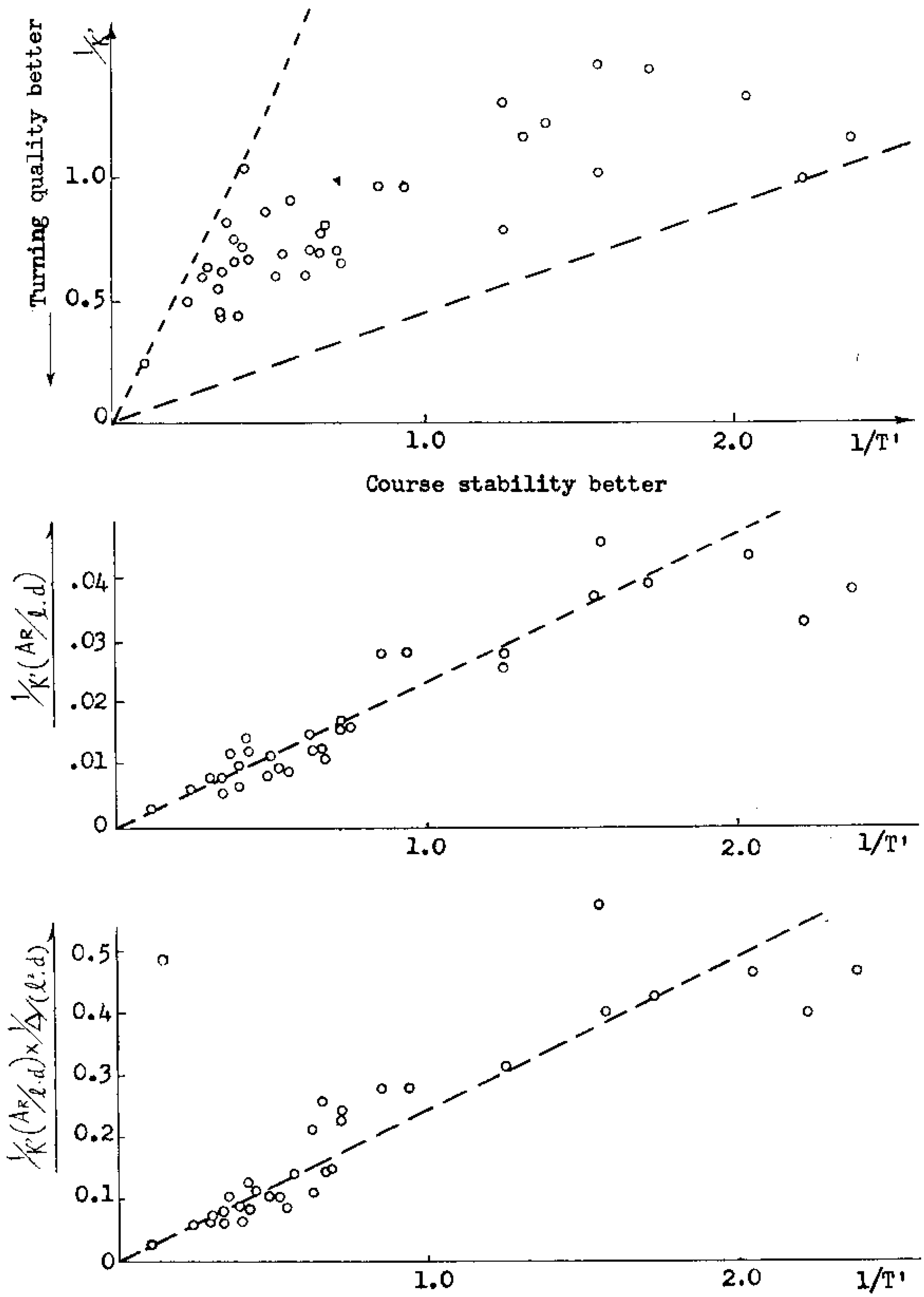
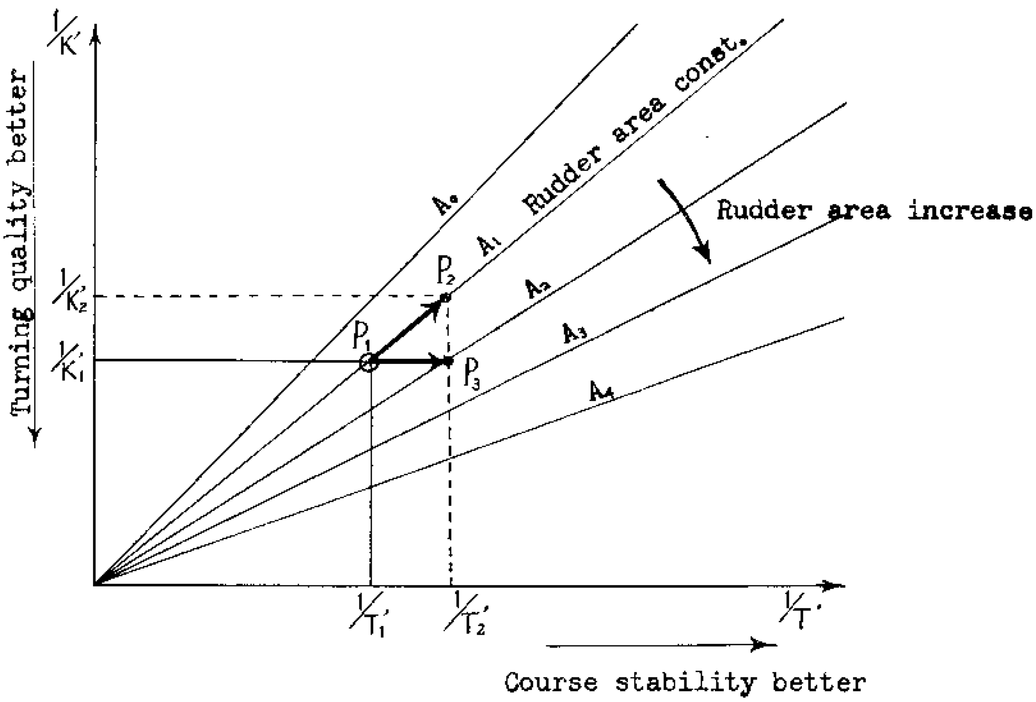



Fig. 14*a*, 14*b*, 14*c*.




tially eliminated by multiplying a coefficient k which is constant irrespective of the ship's form (*).

$$\frac{K}{T} = k \frac{C_\mu}{n} \left(\frac{V}{l}\right)^2 \qquad 19)$$

In conclusion the factor which governs the ratio K/T is the ratio of the rudder moment to the moment of inertia of ships, and accordingly equation (19) can be rewritten as :

$$\frac{K}{T} = \frac{\frac{1}{2}\rho g\, C_n \cos\delta}{(W/l^2 d)\,\delta} \frac{A}{Ld} \frac{\overline{GR}/l}{(k/l)^2} \left(\frac{V}{l}\right)^2 \qquad (20)$$

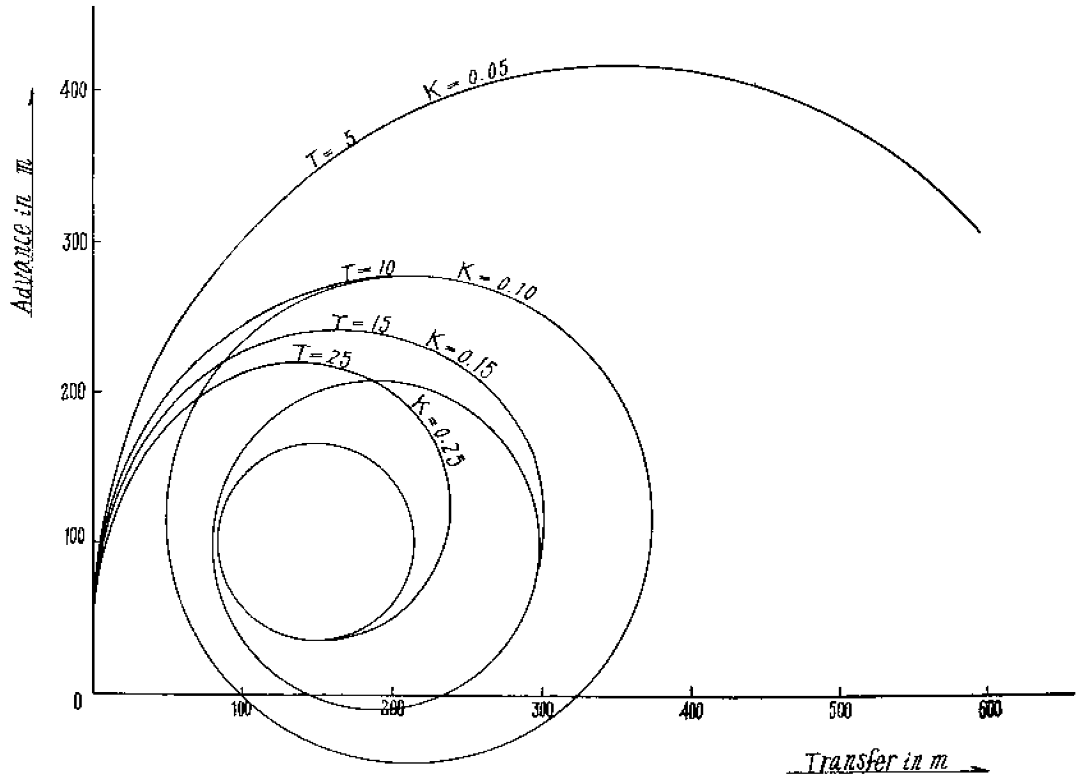
Where : $\rho$ = density of water;
W = displacement of ship;
d = draught;
C = coefficient of normal component of rudder force;
A = rudder area;
GR = distance from the centre of gravity of ship to the centre of pressure of rudder plate;
$k$ = virtual radius of gyration of ship about $x$-axis).

(*) Whilst no assurance has been given to the fact that k does not change according to the ship's form, the comparison with the results of measurements indicates that this assumption is not ill-advised in general. The value of k is approximately 1.25.

Of the above factors governing the ratio K/T, the rudder area ratio $A/l \cdot d$ is the predominant factor, and the effect of other factors, such as $W/l^2\,d$, $k/l$, etc. which are dependent upon the ship's form, are not so great as compared with that of the rudder area ratio. Accordingly it can be deduced that K/T is approximately kept constant unless the rudder area is changed.

The above relation is illustrated in Fig. 15, in which $1/T'$ which represents the course stability or quick responsibility is taken in the abscissa and $1/K'$ which represents the turning ability is taken in the ordinate.

In this figure, the relation given by equation (20) is represented by radial lines intersecting the origin of the coordinate. When the rudder area is varied, these lines shift to radial lines nearer to the base as the rudder area increases. Let us now assume that a ship having a steering quality corresponding to the point $P_1$ in the figure, that is, a ship having the course stability $1/T_1'$ and turning ability $1/K_1'$, is renovated so as to improve the course stability (for instance, by fitting dead wood in the stern). When this renovation is made without changing the rudder area $A_1$, then the ratio K/T does not change, and the point $P_1$ shifts to $P_2$ on the same radial line. In this case, therefore, the course stability becomes inevitably worse. If the improvement of the course stability without adversely




affecting the turning ability is desired, it is necessary to increase the rudder area, since such improvement should be made so as to shift the point $P_1$ to $P_3$ as shown in the figure.

As explained above, the course stability and turning ability have the character which is contradictory to each other, and therefore, in actual ships, the ratio K/T can change only within a comparatively small range. Accordingly the combination of K and T as extreme as given in Fig. 10 cannot exist in actual ships.

Figure 16 shows the turning paths corresponding to various combinations of K and T when the ratio K/T is kept constant. This figure demonstrates that the greater the value of K, that is, the better the turning ability, the better the results for the steering quality as a whole.

Since $K\delta/T$ represents the angular acceleration of turning motion immediately after the steering as indicated by equation (8) and (9), $K/T =$ constant means that the initial angular acceleration is constant so long as the helm angle is unchanged. Accordingly, if the steering efficiency of rudder is considered to be proportional to the initial angular acceleration of turning motion, it can be concluded that the *only way to improve the steering efficiency of rudder is to increase the rudder area.*

References

[1] Davidson K. & Schiff L. : *Turning & Course Keeping Qualities of Ships*, S.N.A.M.E., Vol. 54, 1946.

[2] Nomoto K. : *On the Manoeuvrability of Ships* (1). Journal of the Society of Naval Architects of Japan (J.S.N.A.), Vol. 99, 1956.

[3] Nomoto K. : *On the Manoeuvrability of Ships* (2) J.S.N.A., Vol. 101, 1957.

[4] Takahashi K. & Kawashima E. : *On the Analysis of the Manoeuvrability by Z-manoeuvre Test and Its Applications*, J.S.N.A. (Kansai), Vol. 88, 1957.

[5] Nomoto K. : *On the Analysis of Z-manoeuvre Tests* J.S.N.A. (Kansai), Vol. 92, 1958.

[6] Nomoto K. : *Model Experiments on the Manoeuvrability of Large Oil Tankers*, J.S.N.A., Vol. 103, 1958.

[7] Motora S. : *On the Manoeuvrability Index of Ships*, J.S.N.A., Vol. 104, 1958.

[8] Akazaki S. : *On the Turning Qualities of Ships*, J.S.N.A., Vol. 61, 1937.

[9] Nomoto K. : *Analysis of the Standard Manoeuvre Test of Kempf and proposed Steering Quality Indices*. Proc. 1st Symp. on the Manoeuvrability of Ships. D.T.M.B. 1960.