6. On Wave Excitationless Ship Forms

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Introduction

It has been deduced theoretically by Ursell[1] and Hishida[2] that prisms of certain sectional shapes create no waves when they roll in a still water surface, and experimental check has also been made by McLeod and Hsieh[3]. Bessho has extended this theme into motions of six degree of freedom and developed the theory of "Wave-free distributions"[4].

In addition, Newman[5] has shown, on the basis of Haskind relation, that the amplitude of radiated wave by an oscillating body in a free surface is directly related to the exciting force acting on the same body in waves.

These results indicate that there must be bodies which are free from exciting forces in waves.

The authors, being interested in the possible existence of such wave-excitationless bodies, have been carrying out experimental research for such bodies, and have found that there were a group of bodies which are free from wave-induced heaving force in waves of specified frequencies.

1. Approximate formula of heaving force

Froude-Krylov hypothesis has long been used in estimating wave induced heaving force until body-wave interaction was clarified in recent years. According to the Froude-Krylov hypothesis, heaving force becomes nil only when the waterplane area is zero. However, the authors were aware that body-wave interaction is reverse in sign of that of Froude-Krylov force and that there must be a possibility that a body could be designed to have greater body-wave interaction in relation to the Froude-Krylov force so that they will cancel each other.

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As Motora has shown[6], the heaving force is approximately expressed as a summation of an inertia term, a damping term and a buoyance term as shown in equation (1).

$$F_{z_w}=\gamma_1 k_z \rho V \ddot{z}_w+\gamma_2 N_z \dot{z}_w+\gamma_3 \rho g A z_w \qquad (1)$$

where k_z : added mass coefficient
ρ : density of surrounding fluid
V : volume of a body
z_w : wave elevation
N_z : damping coefficient
g : acceleration of gravity
A : waterplane area
and $\gamma_1, \gamma_2, \gamma_3$ are correction factors for the orbital motion of the wave.

The first two terms correspond to the body-wave interaction, and the third term corresponds to the Froude-Krylov buoyancy. The first term is also an inertia term due to added mass effect. γ_3 has been known as Smith Correction Factor.

If the wave elevation is of the form

$$z_w=ze^{i\omega t}$$

where ω is the circular frequency, then,

$$\ddot{z}_w=-\omega^2 z_w .$$

Therefore, equation (1) is rewritten as follows:

$$F_{z_w}=-\gamma_1 k_z \rho V \omega^2 z_w+\gamma_2 N_z \dot{z}_w+\gamma_3 \rho g A z_w \quad (2)$$

The inertia term is reversed in sign in relation to the buoyancy term and will increase rapidly as the frequency ω increases. In case of ordinary ship form however, the first term is not large enough to cancel the buoyancy term in a frequency range of ordinary wave encounter. However, if the form of a body is chosen so that the inertia term is large enough compared to the buoyancy term, it will be possible to eliminate the heaving force in a relatively low frequency range.

According to Newman[5], heaving force is related to the damping coefficient by the following formulae:

$$\left.\begin{aligned} N_z &= \frac{\omega}{\rho g^2 h^2} F_{z_w}{}^2 \quad \text{(two dimensional case)} \\ N_z &= \frac{\omega K}{\pi \rho g^2 A^2} \int_0^{2\pi} F_{z_w}(\theta) d\theta \quad \text{(three dimensional case)} \end{aligned}\right\} \tag{3}$$

where K is wave number

θ is encounter angle of an incident wave

h is the wave height.

Substitute (3) into (2), we can get approximate value of the amplitude of the heaving force $\bar{F}_{z_w}$ as follows:

$$\bar{F}_{z_w} \doteqdot (-\gamma_1 k_z \rho V \omega^2 + \gamma_3 \rho g A)\bar{z}_w \sqrt{1 + (\gamma_1 k_z \rho V \omega^2 + \gamma_3 \rho g A)^2 \left(\frac{\gamma_2 \omega^2}{\rho g^2 h^2}\right)^2 \bar{z}_w{}^4} \ . \tag{4}$$

Therefore, heaving force will vanish when the frequency ω takes the following value:

$$\omega_0 = \sqrt{\frac{\gamma_3 g A}{\gamma_1 k_z V}} \tag{5}$$

or

$$K_0 = \frac{\gamma_3 A}{\gamma_1 k_z V} \, . \tag{6}$$

Let us call ω_0 as "excitationless frequency" for convenience. To bring this excitationless frequency towards low frequency range of probable wave encounter, it will be necessary to make A/V smaller than usual proportion. It will be noted that results of measured external force to oscillate different bow section models by Paulling[7] indicate the similar tendency though, in his case, the inertia force includes the mass of a model itself.

2. Two dimensional case

2.1 *Submerged circular cylinder with a strut*

1) Theoretical consideration

In seeking a body which has relatively greater inertia force, one may aware that an extreme case of such body is a completely submerged body. The main part of the heaving force acting to a submerged body is an inertia force which is reverse in sign of the wave elevation. Therefore, if a vertical strut of narrower width is attached to a submerged body so that it gives small amount of vuoyancy, it will be possible to eliminate the heaving force at a specified frequency of waves.

Therefore, let us choose a combination of a circular cylinder of radius a, depth f, and a vertical strust of breadth B as shown in Fig. 1.

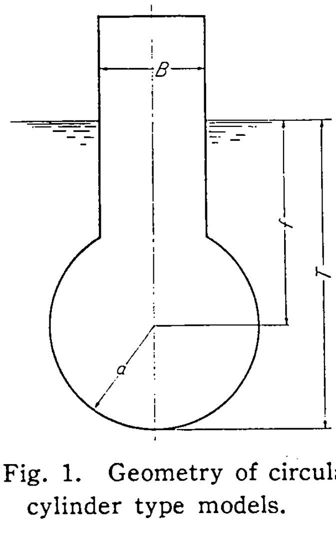

Fig. 1. Geometry of circular cylinder type models.

Though the heaving force acting on a circular cylinder has been calculated by Cummins[8], it is necessary for us to calculate the inertia term and buoyancy term separately, an approximate calculation was done making use of the equation (4), i.e., it was assumed approximately that a summation of the inertia term for a a cylinder and for a strut would give the inertia term of the total body. The same assumption was applied for the buoyancy term.

For the inertia coefficient k_z of a circular cylinder, values given by Yamamoto[9] was used, and for that of a strut, values given by Tasai[10] for full section was employed.

Calculated inertia force and buoyancy for different breadth of the strut are as shown in Fig. 2.

The approximate values of the heaving

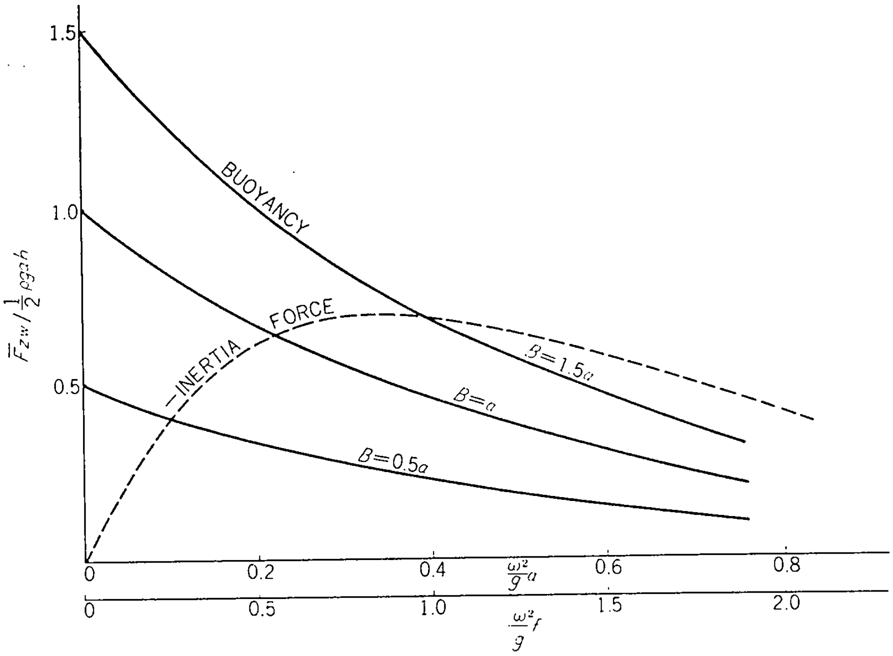

Fig. 2. Calculated inertia force and buoyancy for different breadth of the strut.

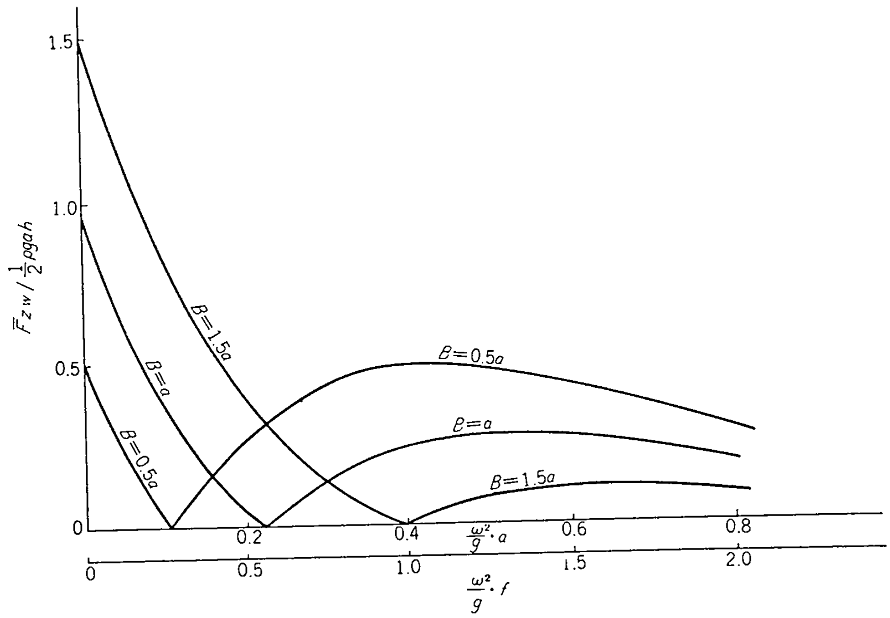

Fig. 3. Calculated heaving force of circular cylinder type models.

force is obtained by equation (4) as shown in Fig. 3.

At relatively low frequency range, the buoyancy is greater than the inertia force (or under cancel), and the heaving force will be in-phase of the wave elevation. At ω_0 given by the equation (5), heaving force is zero, and at higher frequency than ω_0, the inertia force over cancels the buoyancy, i.e., the heaving force at this frequency range will be 180° out-of-phase of the wave elevation.

It is also shown in Fig. 3 that ω_0 shifts toward lower frequency when the breadth of the strut decreases.

2) To check the above described results, experiments were conducted on a model of the following size.

Diameter $2a=20$ cm

Breadth of the strut $B=15$ cm ($1.5a$), 10 cm (a), and 5 cm ($0.5a$)

Length=50 cm (with end plates).

To prevent the sway and pitch of the model a guide is attached to the model as shown in Fig. 4. The heaving force was measured by a rigid spring attached by a linear transformer as a pick-up.

A typical result for $B=a$ and depth $f=2a$ is shown in Fig. 5 together with the computed one. The base of Fig. 5 is non dimensionalized wave frequency and the ordinate is the non dimensionalized heaving force. Thick solid line shows the heaving force of a semi-submerged circular cylinder of radius $a/2$.

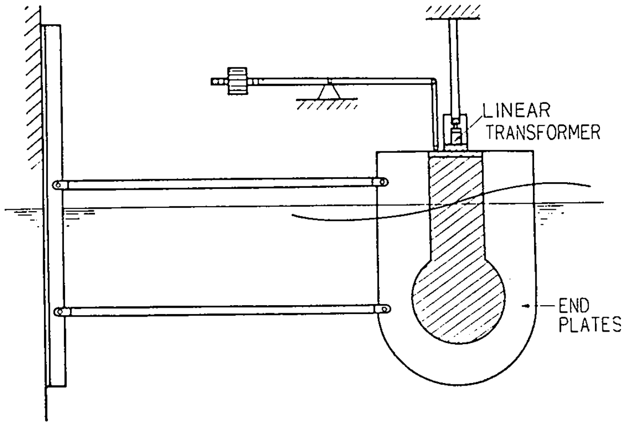

Fig. 4. Set-up of heaving force measurement.

A remarkable decrease of the heaving force at a frequency approximately equal to the theoretical ω_0 will be recognized.

Fig. 6 is typical oscillogram of the heaving force and the wave elevation as $(\omega^2/g)a=0.124$, 0.238 and 0.402. At $(\omega^2/g)a=0.124$, where the buoyancy is greater than the inertia force, the heaving force is in-phase of the wave elevation (under cancel). At $(\omega^2/$

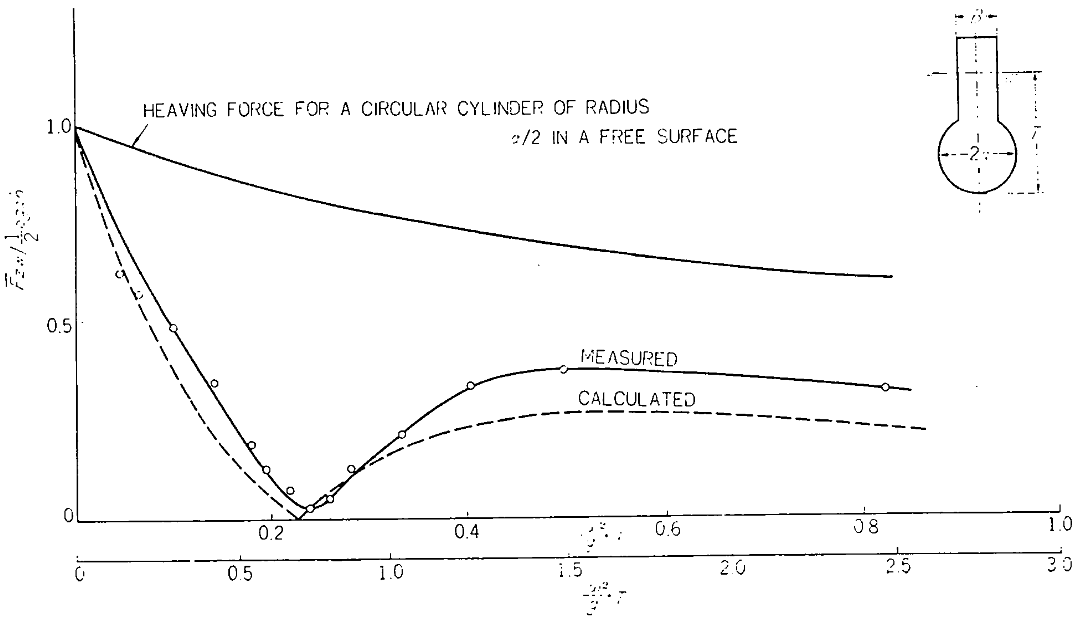

Fig. 5. Typical result.

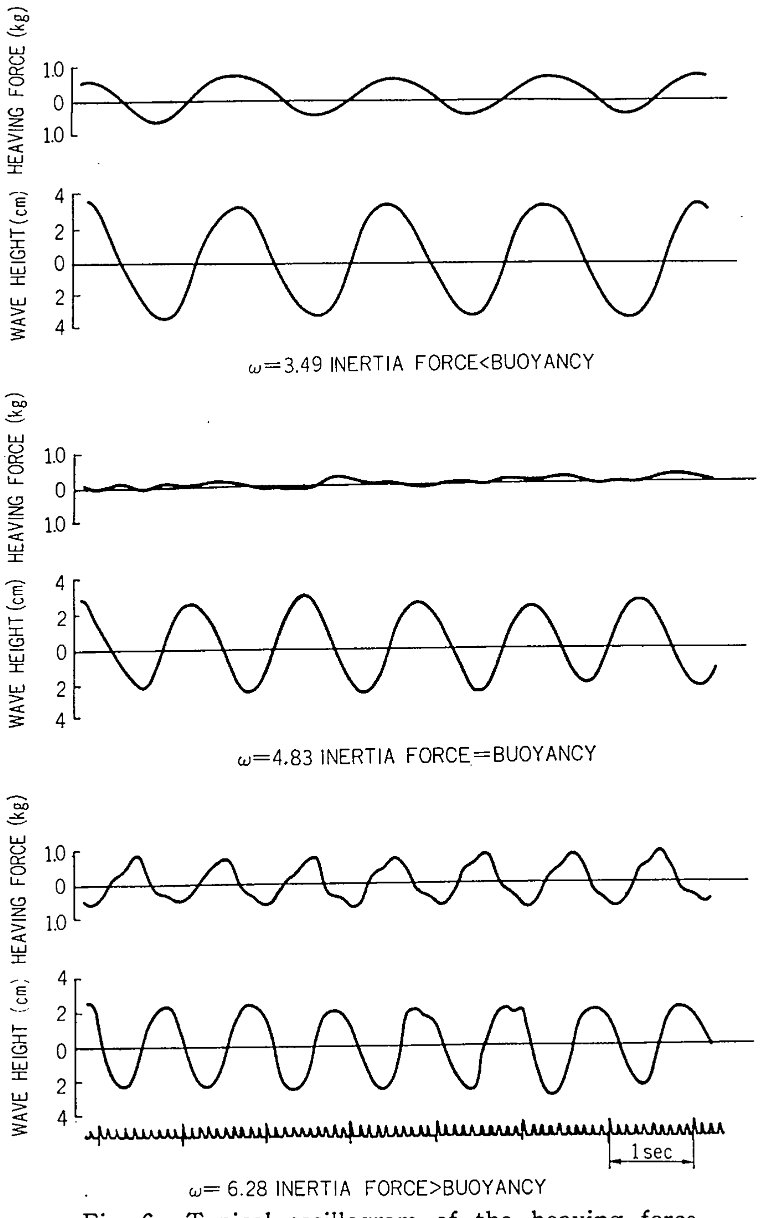

Fig. 6. Typical oscillogram of the heaving force and wave elevation.

$g)a=0.238$, the heaving force almost vanishes. At $(\omega^2/g)a=0.402$, where the buoyancy is less than the inertia force, the heaving force is 180° out-phase of the wave elevation (over cancel). These oscillogram gave valuable information in determining a suitable breadth of the strut.

To examine the effect of the breadth of the strut as well as the depth of the cylinder, results are shown in Fig. 7 a), b), c). As predicted by theory, ω_0 shifts to lower frequency as the breadth of the strut decreases. However, change of depth does not affect ω_0. ω_0 for $a=10$ cm, $T=30$ cm is 5.03 which corresponds to $\omega_0=0.873$ for a full scale ship of length 140 m and draft 10 m. This frequency corresponds to wave length 61 m for beam seas. However, in case of longitudinal waves, this frequency corresponds to wave length 200 m at ship speed 18 kts.

a)

b)

c)

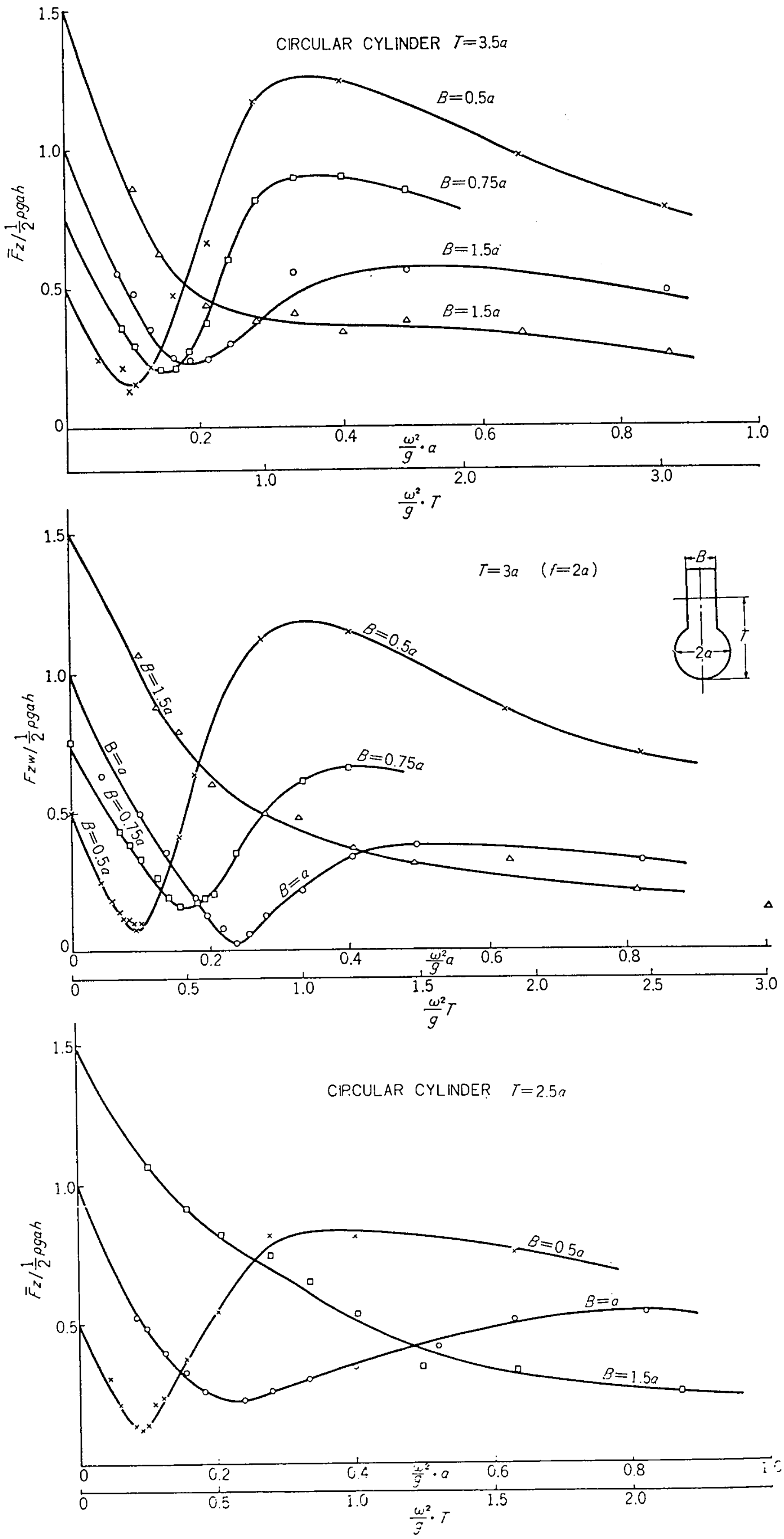

Fig. 7. Heaaving force of the circular cylinder type models.

2.2 *Elliptic cylinder with strut*

To make the draft relatively small, elliptic cylinders were chosen as the main body. The proportion of major axis and minor axis were varied from 2 to 4, where the breadth of the strut was kept to be a half of the major axis. (See Fig. 8)

Results are shown in Fig. 9 a) and b). Different from the result of the former case, ω_0 seems to shift towards lower frequency as the depth increases. Absolute value of the heaving force is about the same as the former case.

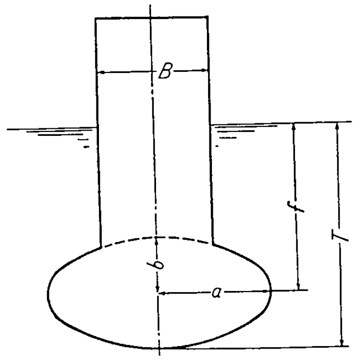

Fig. 8. Geometry of elliptic cylinder type models.

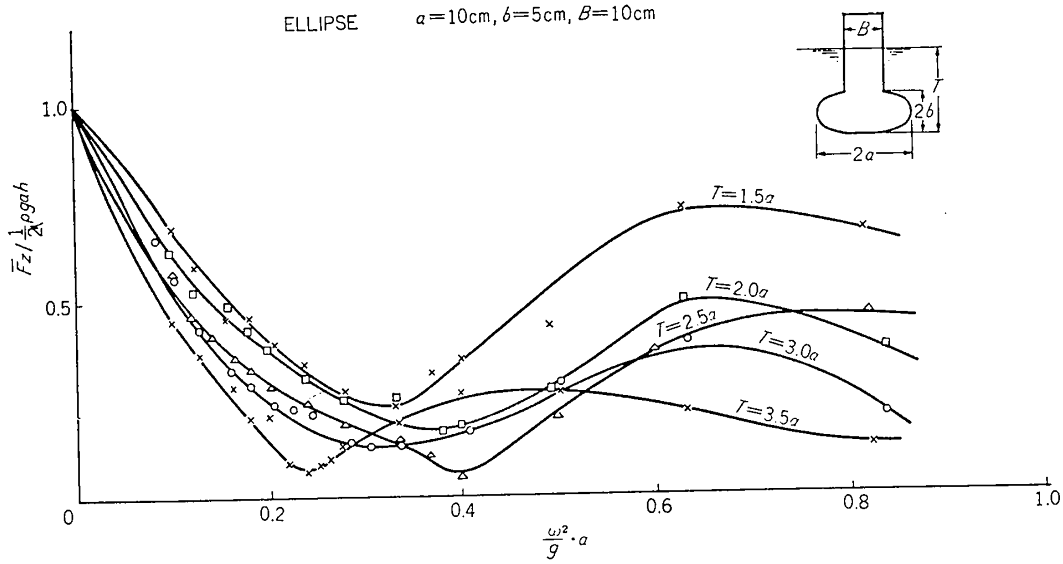

Fig. 9 a). Heaving force of elliptic cylinder type ($a/b=2$).

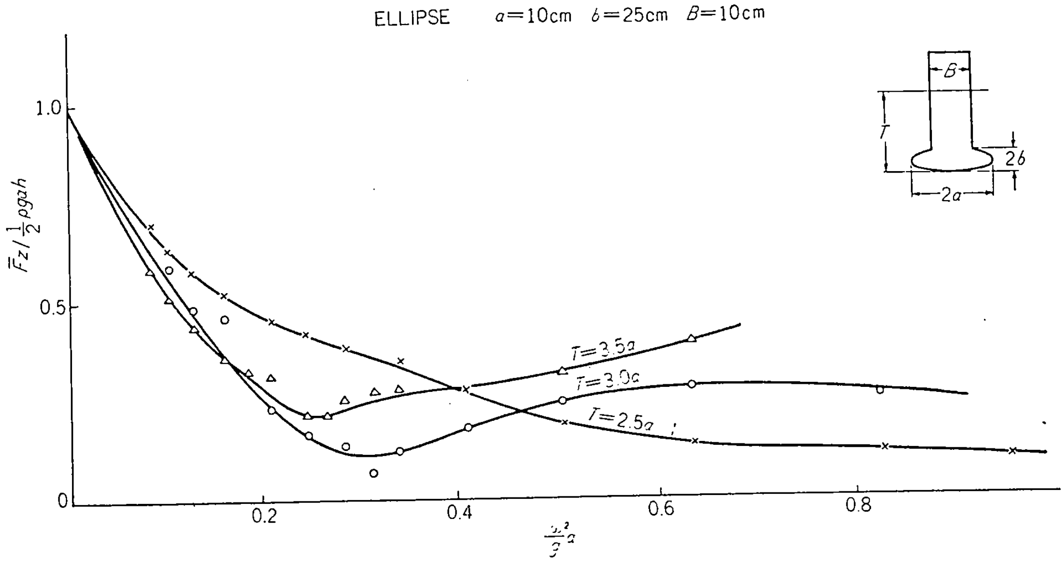

Fig. 9 b). Heaving force of elliptic cylinder type ($a/b=4$).

2.3 *Fins with strut*

To make the inertia term greater, it is necessary to increase the added mass $k_z\rho V$. The virtual mass for a thin plate of breadth $2a$ is $\rho\pi a^2$ per unit length. Therefore, it will be possible to replace ellipse by adequate size of fins as shown in Fig. 10.

A model in which $2a=25$ cm, $B=15$ cm, $f'=7$ cm was tested in the same technique. Results are as shown in Fig. 11. In general, it can be said that effectiveness of fins are almost equivalent to a thin ellipse.

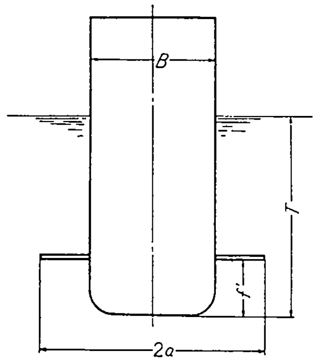

Fig. 10. Geometry of fin type models.

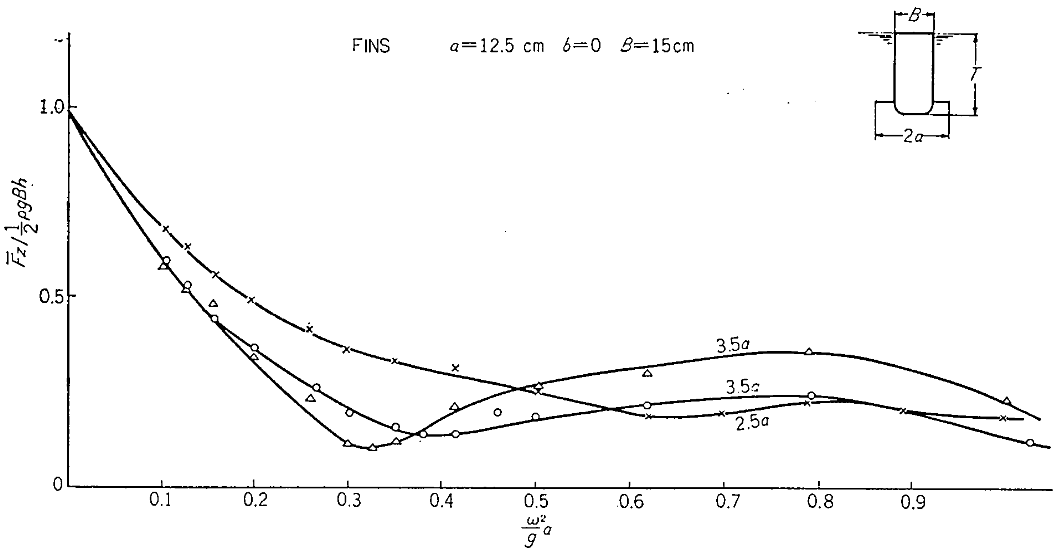

Fig. 11. Heaving force of fin type models.

3. Heaving amplitude

As shown in Section 2, there are several variations of bodies which are not acted by wave induced heaving force at specified frequency. However, it will be premature to conclude that these bodies do not heave at the specified frequency.

As shown in equation (3), damping factor is proportional to the square of the heaving force. This means that wave excitationless body is also a dampingless body.

In fact, in the case of the circular cylinder as described in 2) of Section 2.1, measured heaving amplitude is quite large as shown in Fig. 12. There appears a high peak at resonant frequency and a minimum point at a frequency about ω_0. It will be interesting to bring the resonant frequency equal to ω_0. However this is proved not to be practicable by the following reason:

resonant frequency of heaving

$$\omega_s=\sqrt{\frac{\rho g A}{m+m_z}} \tag{7}$$

where m is the mass of a body.

excitationless frequency

$$\omega_0=\sqrt{\frac{\gamma_3 g A}{\gamma_1 m_z}}\,. \tag{8}$$

As the ratio γ_3/γ_1 is almost equal to the unity, ω_s can not be equal to ω_0 unless the mass of the body is zero.

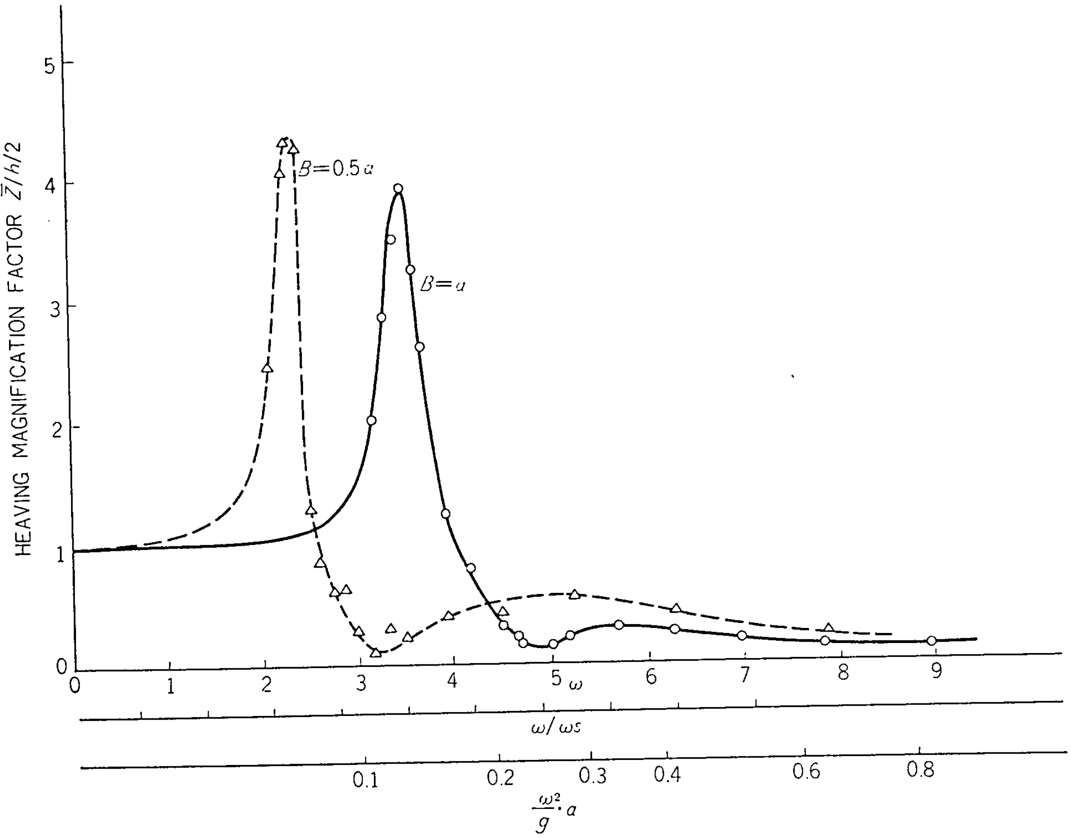

Fig. 12. Heaving magnification factor of circular cylinder type model.

As far as the wave damping is concerned, it seems hopeless to decrease the heaving amplitude. However, it should be noted that eddy damping or artificial damping are not included in Haskind-Newman relation.

Therefore, eddy making damping or artificial damping such as by a passive tank are given to a wave excitationless body, it will be possible to minimize the heaving.

Heaving of a ellipse ($a/b=2$) with a strut is measured as shown in Fig. 13 a) in which less heaving amplitude than the case of a circular cylinder due to eddy damping will be recognized. Thinner ellipse ($a/b=4$) with a strut is also tested as shown in Fig. 13 b). Remarkable decrease of heaving due to eddy damping will be noted.

Heaving of a body with fins as described in 2.3 is also tested. Results are as shown in Fig. 14. Though the magnification factor diagram looks like that of critical damping, actual damping is about one third the critical damping. Apparent critical damping of the magnification factor diagram is due to rapid decrease of the exciting force as the frequency approaches to ω_0.

4. A trial to eliminate wave excitation in wider range of frequency

According to the above described method, the exciting force is eliminated at only one specified frequency. The following is a trial to eliminate the exciting force at two or more frequencies.

As shown in Fig. 15, a inner solid strut of breadth B_1 is covered by a tank of breadth B_2. The tank is provided by small holes of total area S, depth H.

If we denote the excitationless frequency for B_1 as ω_{01}, the heaving force will vanish at frequency ω_{01} provided the area S is chosen adequately so that the water level in the tank is always equal to that of outer surface at this frequency.

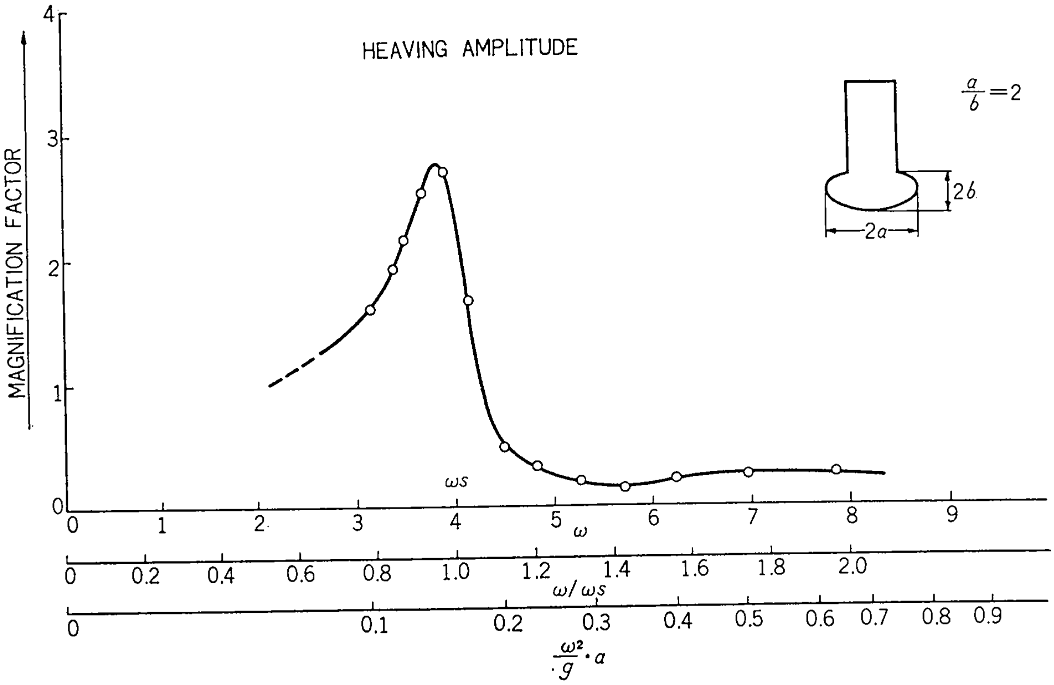

Fig. 13 a). Heaving magnification factor of an elliptic cylinder type model ($a/b=2$).

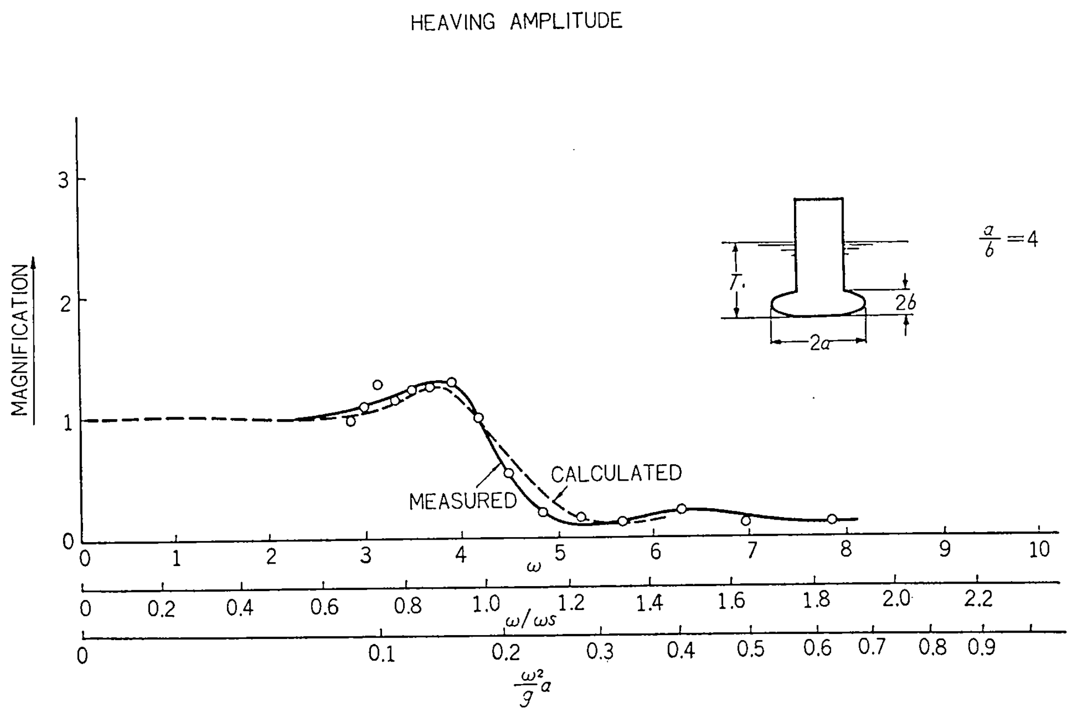

Fig. 13 b). Heaving magnification factor of an elliptic cylinder type model ($a/b=4$).

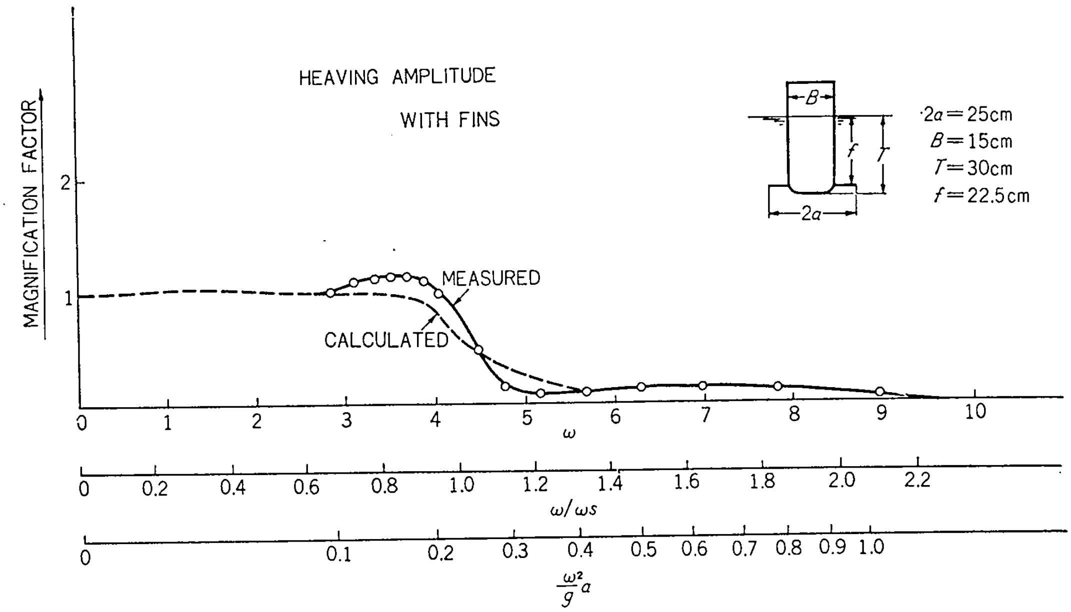

Fig. 14. Heaving magnification factor of a fin-type model.

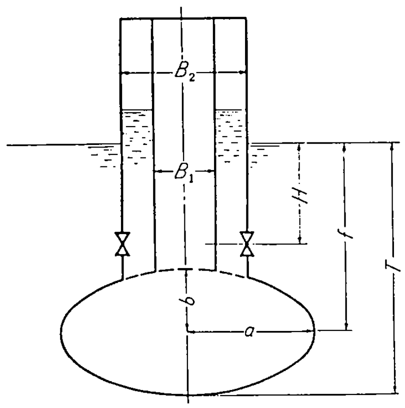

Fig. 15. Geometry of double caisson type models.

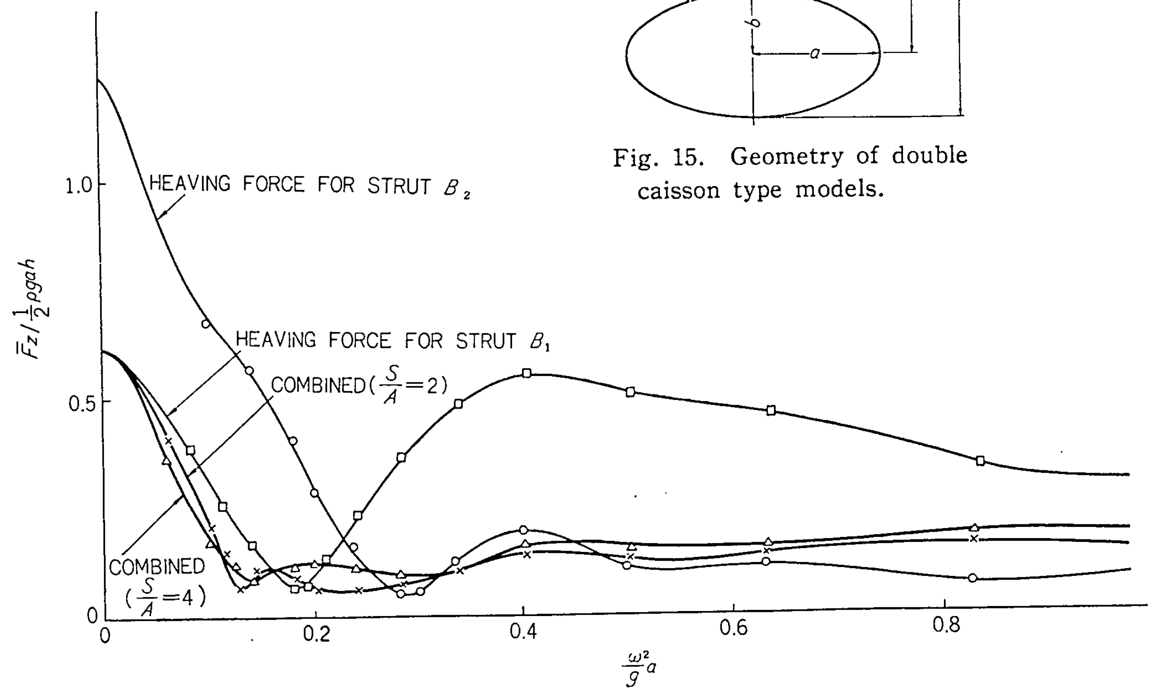

Fig. 16. Heaving force of a double caisson type model.

On the other hand, according to a theory of anti-pitching tank[12),13)], the movement of water in the tank, as the outer water level changes periodically, becomes zero when the frequency coincides to the following frequency;

$$\omega_0'=\sqrt{\frac{g}{H}}\ .$$

Therefore, at the frequency ω_0', tank water is practically solidified and the breadth of the strut is practically B_2.

Therefore, if the excitationless frequency for B_2 is denoted ω_{02}, heaving force will vanish at ω_{02}. Thus, the heaving force will vanish at two different frequencies ω_{01}, and ω_{02}.

Experiments were conducted on a model for which B_1, B_2 and H where chosen as shown in Fig. 15. Results are as shown in Fig. 16 in which three results for different size of the holes are indicated. By Fig. 16, it will be easily seen that the heaving force almost vanishes at predicted ω_{01}, and ω_{02}, and is very small in a frequency range founded by ω_{01} and ω_{02}.

5. Three dimensional problems

5.1 *A sphere with vertical cylinder*

As a most simple three dimensional case, let us consider a sphere with vertical cylinder as shown in Fig. 17. As the equation (2) is applicable for three dimensional case provided adequate values for γ are chosen, the amplitude of heaving force of such body

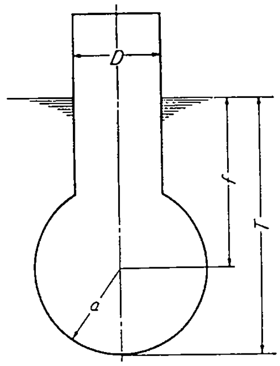

Fig. 17. Geometry of sphere type models.

is calculated by equation (2) as shown in Fig. 18.

The heaving force is also measured of a model of the following size;

$$a=14.3\,\text{cm}\qquad D=a\ \text{and}\ \sqrt{2}a\,,$$
$$T=1.5a\ \text{and}\ 2a\,.$$

A typical record obtained is shown in Fig. 19 which shows almost the same tendency as the two dimensional case shown in Fig. 6.

In Fig. 20 a), b), heaving forces for different depth are shown. Reasonable agreement with theoretical calculation will be noticed.

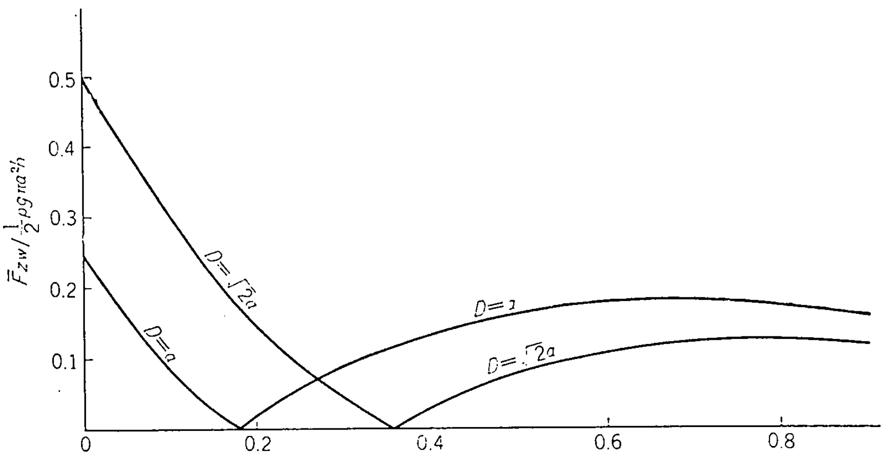

Fig. 18. Calculated heaving force of sphere type models.

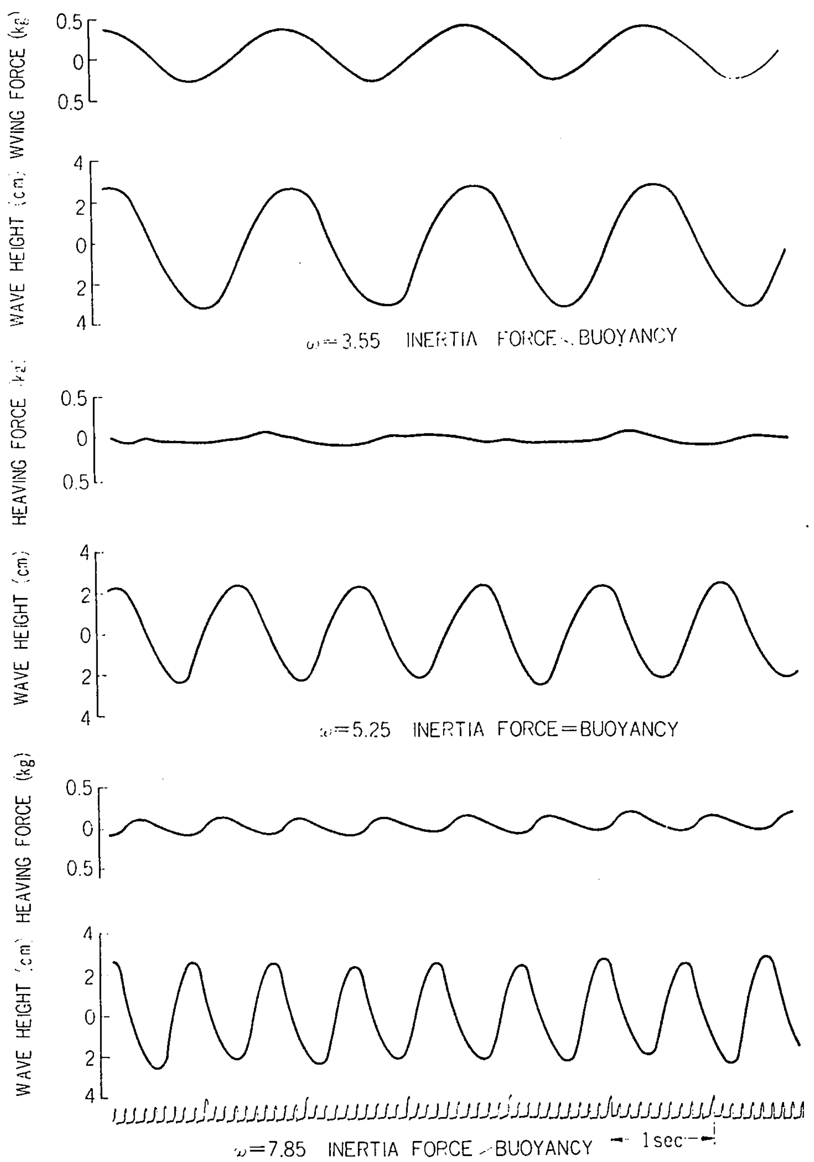

Fig. 19. Typical record obtained for a sphere type model.

5.2 *A prism in longitudinal wave*

In relation to the possibility of applying the strip method for three dimensional cases, it is important to know how a wave act to a section unit length of a prism fixed in longitudinal wave train.

This problem was treated by Okumura and Sugiura[11] for a prism of a section shown in Fig. 13 b). An unit length of prism was cut off and kept free from the main body, and was supported by a rigid spring attached with linear transformer as shown in Fig. 21, so that heaving force acting on this segment was measured.

Results are as shown in Fig. 22. In Fig. 23, heaving forces in beam seas as well as longitudinal wave are compared with theoretical value. Reasonable agreement between them will suggest that the strip method will be applicable for obtaining heaving force as well as pitching moment of ships in longitudinal waves. Therefore, there will be a possibility to eliminate pitching moment of ships by the same technique.

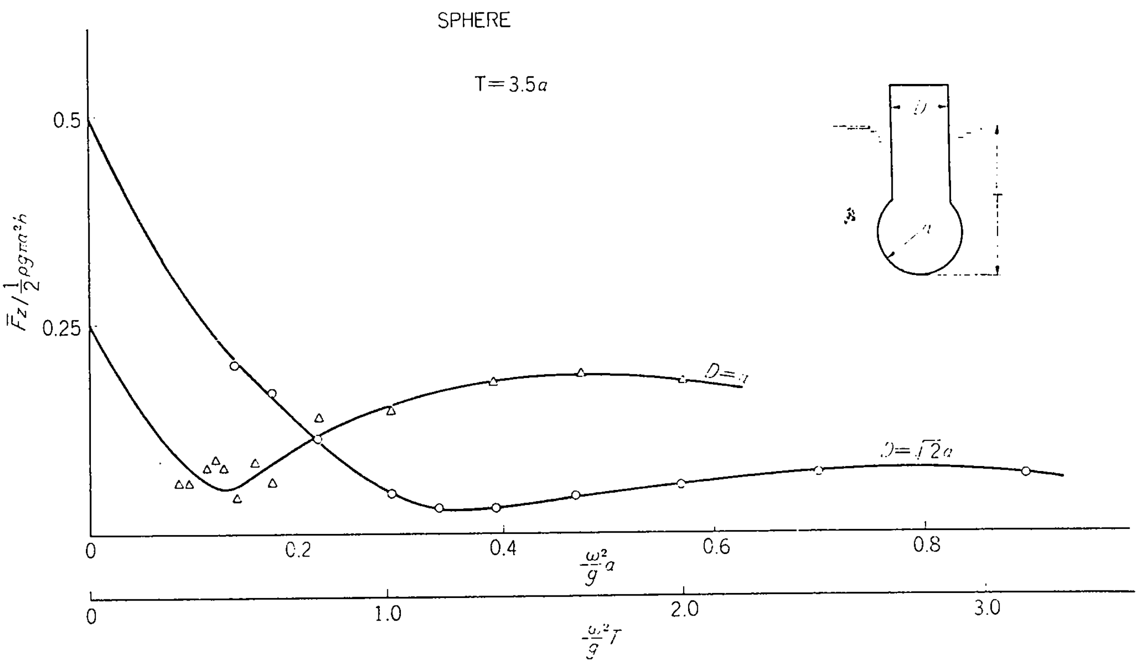

Fig. 20 a). Heaving force of sphere type models (T=3.5a).

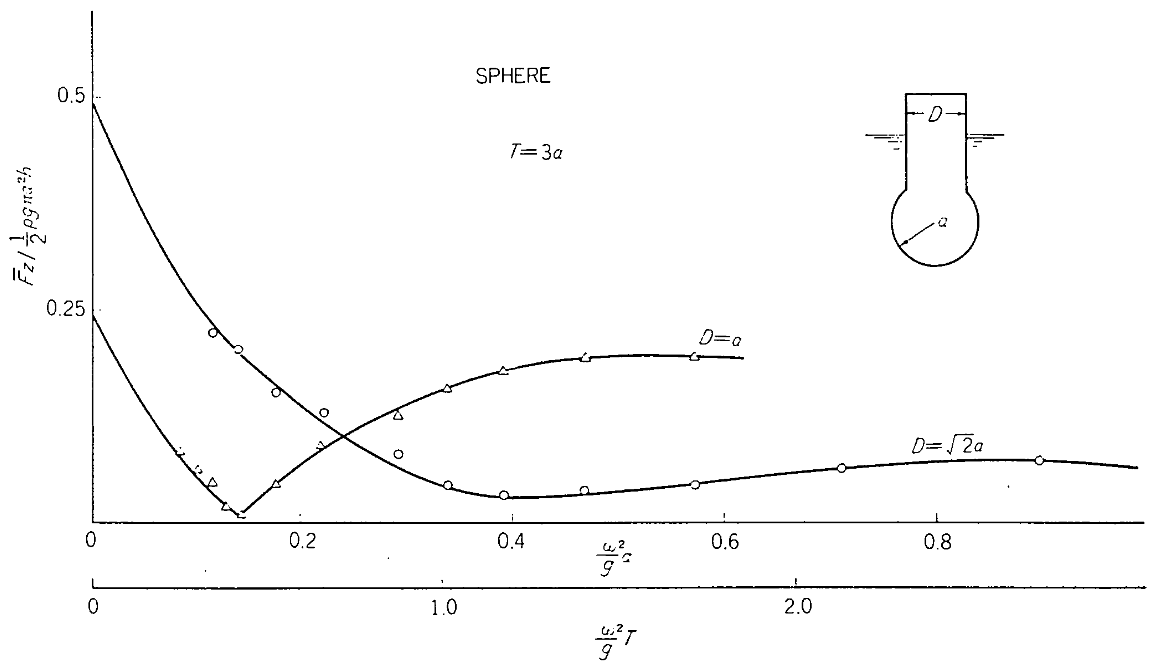

Fig. 20 b). Heaving force of sphere type models (T=3a).

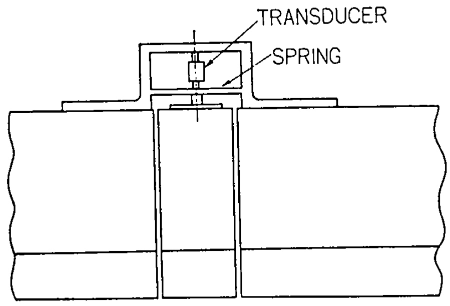

Fig. 21. Set-up of heaving force measurement.

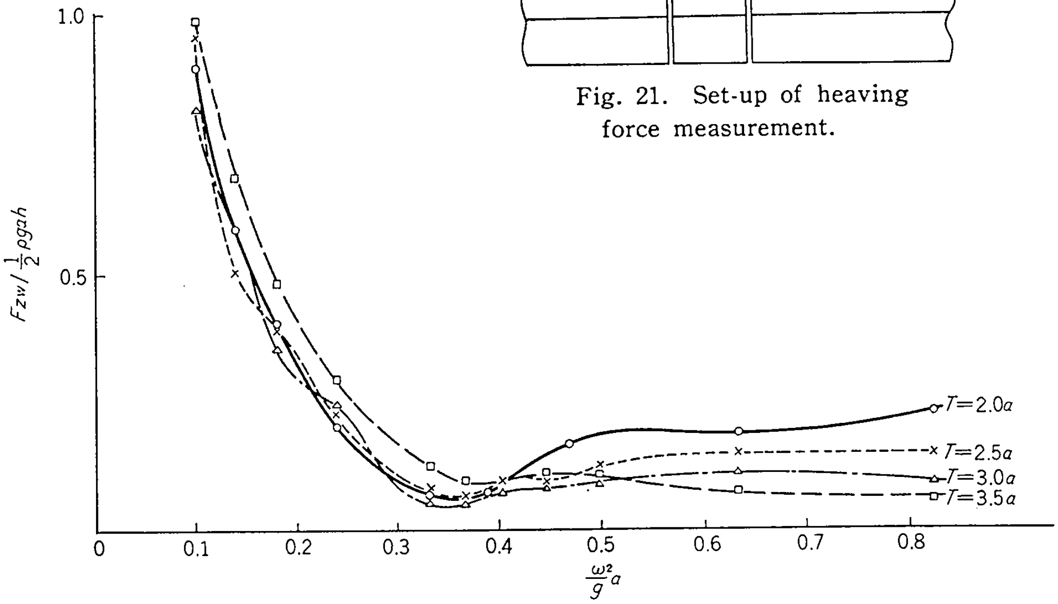

Fig. 22. Heaving force per unit length of an elliptic type prism in longitudinal wave.

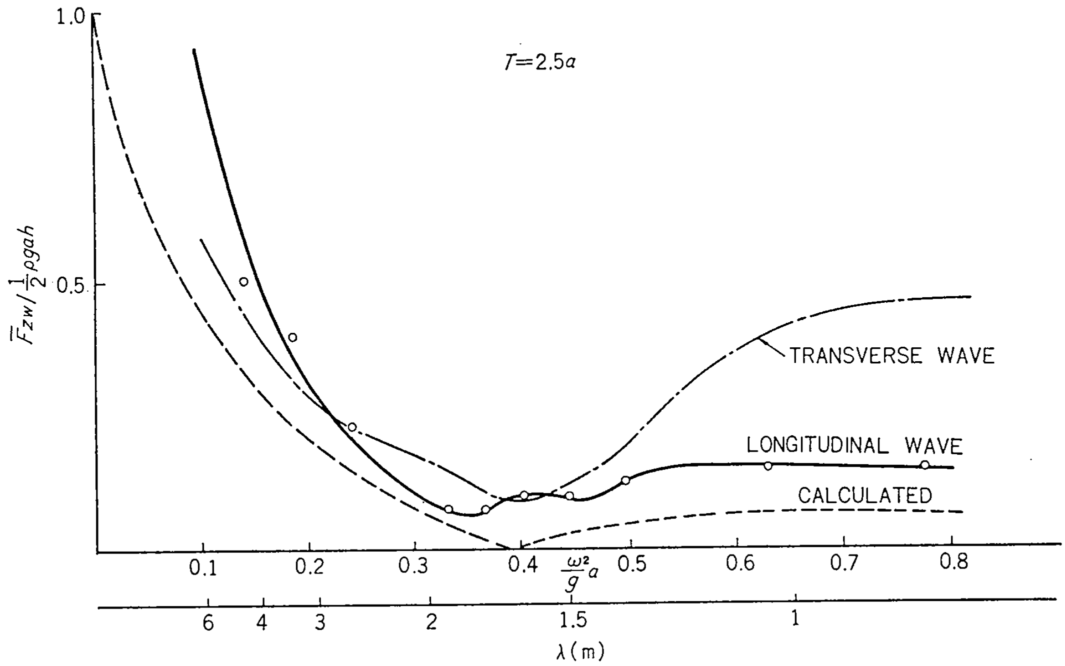

Fig. 23. Comparison of heaving force in longitudinal waves with that in transverse waves.

6. Comparison with the "wave-free distribution" theory

Bessho has given in his paper "wave-free distribution"[4], a group of two dimensional bodies which are wave-free as they heave in a free surface. They are derived by the following procedure:

Supposing $m(t)$ is a function which is regular at infinity and is vertically skew-symmetry. Then a function

$$f(t)=Km(t)+i\frac{d}{dt}m(t) \qquad (10)$$

will give a velocity potential which satisfies the free surface condition, and should not associate surface waves. Therefore, $f(t)$ is a velocity potential around a body which is free from surface wave as it heaves in a free surface.

If a singular point at depth h is taken to be $m(t)$, viz,

$$m(t)=\log\left(\frac{t+ih}{t-ih}\right) \qquad (11)$$

then,

$$f(t)=\frac{i}{t+ih}-\frac{i}{t-ih}+K\log\left(\frac{t+ih}{t-ih}\right) \qquad (12)$$

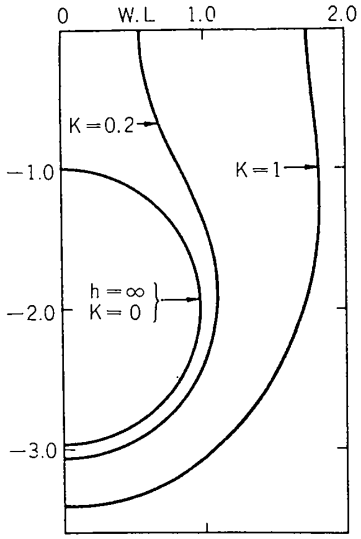

Fig. 24. Typical stream lines of two-dimensional wave-free distributions.

stream lines given by this function are as shown in Fig. 24. As seen in Fig. 24, one of them is similar to the circular cylinder type model of the present paper.

It is quite interesting to see that two different approaches reached similar results.

An experiment was conducted to measure the heaving force for such body. Results are as shown in Fig. 25 where heaving force

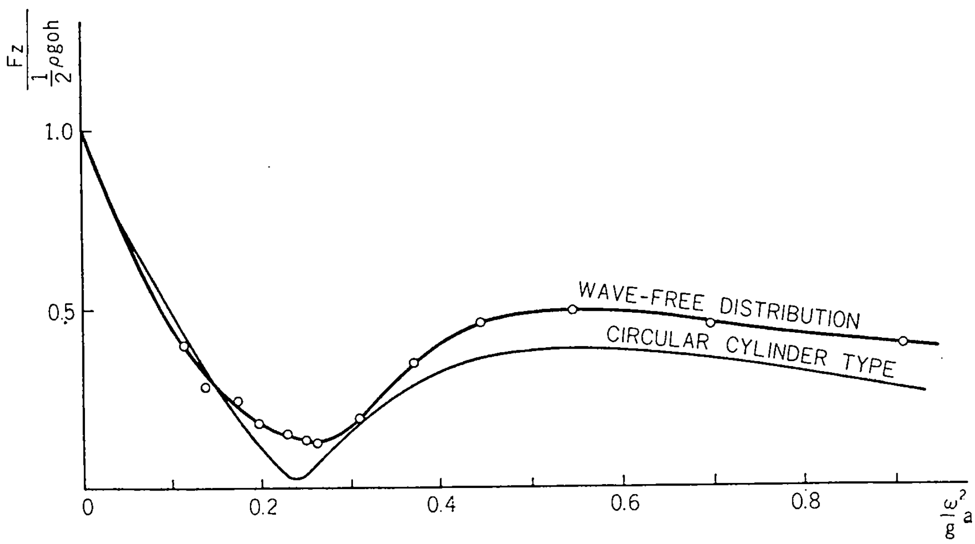

Fig. 25. Comparison between the heaving force of a wave-free body obtained by Bessho and that of the circular cylinder type model.

for a circular cylinder type is also shown for comparison. From Fig. 25, it will be noticed that the wave-free body shows quite similar heaving force vs. frequency curve compared with the circular cylinder type model.

Conclusion

Summarizing foregoings, the following can be concluded;

1) The Froude-Krylov buoyancy and the inertia force which is induced by the added mass effect are reverse in sign. Therefore, it is possible to make them cancel each other so that wave excitation is reduced.

2) At a frequency at which the absolute value of the inertia force is equal to that of buoyancy, the heaving force becomes exactly to be zero theoretically, and is proved experimentally to be almost zero.

3) To bring the excitationless frequency into low frequency range such as ordinary wave encounter frequency, it will be necessary to make the underwater volume large enough in relation to the water plane area. The underwater volume can be replaced by a flat plate which has the same amount of added mass as the underwater body.

4) Wave-damping also vanishes as the heaving force vanishes. Therefore, it will be necessary to give damping other than wave-damping to a body to reduce heaving or pitching motion.

5) In the case of two-dimensional body, for a segment of unit length, heaving force due to beam seas is almost equal to that due to longitudinal wave. Therefore, ship method can be applied for three-dimensional bodies. It will be also possible to apply the present technique to the pitching motion.

6) It is also possible to extend this technique into three-dimensional problem.

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