10. Equivalent Added Mass of Ships in Collisions

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Summary

To calculate the absorbed energy by the flacture of ship's structure in case of collision, it is necessary to estimate accurately the added mass of striken ships.

Minorsky has proposed in his famous paper[1] to use 0.4 for the added mass coefficient of striken ships. In this paper, the authors tried to obtain an exact expression of the hydrodynamic force which will act on ships durig a collision by theoretical calculations and series of model experiments on the first atomic-powered ship of Japan were conducted.

As a result, the authors have shown that the ratio of external force to the acceleration of the striken ship varies with the time elapsed during the collision: i.e., the added mass of the ship is not constant during the collision. From this result, the authors introduced an equivalent added mass which, dividing the external force by it, will give an acceleration equal to the exact value of acceleration at the end of the collision.

It was also shown that this equivalent added mass changes its value with the "duration" of the collision. If the duration is infinitely small, the equivalent added mass is equal to the added mass for infinite high frequency. This agrees with Minorsky's assumption. However, if the duration is finite, the equivalent added mass becomes larger as the duration increases.

Since the duration is only related to the initial speed of the striking ship and the amount of penetration, and lower the initial speed or deeper the penetration the longer the duration, the equivalent added mass coefficient will be much greater than 0.4 in the case when a soft structured ship is striken by a low speed ship resulting in a considerable amount of penetration.

1. Introduction

It is necessary to take into account of hydrodynamic effect of the surrounding water in dealing with the absorbed energy by collision of two ships. Minorsky[1], in his famous paper, has equated the absorbed energy and the loss of the kinetic energy of ships involved in the collision. He has also assumed that the hydrodynamic effect was represented by an increased inertia force due to added mass effect and estimated the added mass coefficient of the collided ship to be 0.4 which was used by Koch[2], Dieudonné[3] and Johnson[4] as the added mass coefficient for lateral vibration. In this apper, the authors treated the problem by hydrodynamic point of view. Since the hydrodynamic force in a transient condition such as a collision is function of time as well as acceleration, it is not possible to express the hydrodynamic force as a simple product of an added mass (a constant) and the acceleration. The authors proposed to introduce an equivalent added mass i.e., supposing an exact solution of the acceleration "a" of the collided ship acted by a prescribed force F, then $\bar{m}''$ given by the following equation is defined to be the equivalent added mass for the acceleration.

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$$F=(m+\bar{m}'')a$$

where m is the mass of the ship itself.

In the same sense, an equivalent added mass for the speed which gives an exact final speed, and an equivalent added mass for the absorbed energy can be defined.

2. Theoretical calculation of the equivalent added mass in a collision

As the apparent mass and damping of a ship under periodic motion are functions of the circular frequency ω, let us denote these values as $m(\omega)$ and $c(\omega)$. Then the equation of motion of a ship for transverse motion $x(t)$ under effect of external force $f(t)$ is written down as follows

$$m(\omega)\ddot{x}(t)+c(\omega)\dot{x}(t)=f(t) \tag{1}$$

The Fourier transform of $x(t)$, $f(t)$, denoted by $X(\omega)$, $F(\omega)$ will be given in the following equation.

$$X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{2}$$

The item we are aiming to obtain is $\ddot{x}(t)(\equiv a(t))$, let us obtain $A(\omega)$ which is the Fourier transform of $a(t)$.

Since $A(\omega)=(j\omega)^2X(\omega)$, we get

$$A(\omega)\equiv H(\omega)\cdot F(\omega)$$
$$H(\omega)=\frac{j\omega}{j\omega m(\omega)+c(\omega)} \tag{3}$$

$a(t)$ can be obtained by the inverse Fourier transform of $A(\omega)$

$$a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)e^{j\omega t}d\omega \tag{4}$$

In general, $a(t)$ can be obtained either from the real part $R(\omega)$ or imaginary part $I(\omega)$ of $A(\omega)$.

$$a(t)=\frac{1}{\pi}\int_{-\infty}^{\infty} R(\omega)\cos\omega t d\omega=-\frac{1}{\pi}\int_{-\infty}^{\infty} I(\omega)\sin\omega t d\omega$$
$$A(\omega)=R(\omega)+jI(\omega) \tag{5}$$

a) In case of a step input

Let us first consider an external force which is a step function $U(t)$ concerning the time

$$f(t)=U(t)=\begin{cases}0, & t\geq 0\\ 1, & t<0\end{cases} \tag{6}$$

The Fourier transform $F(\omega)$ of $U(t)$ is expressed as follows using Dirac's δ function.

$$F(\omega)=\pi\delta(\omega)+\frac{1}{j\omega} \tag{7}$$

Then $R(\omega)$ and $I(\omega)$ of eq. (5) are given as follows

$$\left.\begin{aligned}R(\omega)&=\frac{\pi m(\omega)\omega^2\delta(\omega)+c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\\ I(\omega)&=\frac{\pi c(\omega)\omega\delta(\omega)-m(\omega)\omega}{m^2(\omega)\omega^2+c^2(\omega)}\end{aligned}\right\} \tag{8}$$

Substituting the first equation of (8) into (5) we get,

$$a(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\pi m(\omega)\omega^2\delta(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cos\omega t d\omega+\frac{2}{\pi}\int_{0}^{\infty}\frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cos\omega t d\omega \tag{9}$$

where at the 2nd term, the range of integral is converted $(0, \infty)$ considering the integrand to be an even function. According to Tasai[5], $c(\omega)$ is of order of ω^5 when ω is sufficiently small, the 1st term can be evaluated as follows

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\pi m(\omega)\omega^2\delta(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cos\omega t d\omega=\frac{1}{m(0)} \tag{10}$$

where $m(0)$ is the apparent mass at zero frequency. Thus we obtain

$$a(t)=\frac{1}{m(0)}+\frac{2}{\pi}\int_{0}^{\infty}\frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cos\omega t d\omega$$

As a reference, let us examine the initial value $a(0)$ of $a(t)$. The initial value $a(0^+)$ of $a(t)$ will be obtained from $A(\omega)$ as follows provided $a(t)$ does not include any impulse function

$$a(0^+)=\lim_{\omega\to\infty}[j\omega A(\omega)] \tag{12}$$

On the other hand

$$\lim_{\omega\to\infty}[j\omega A(\omega)]=\lim_{\omega\to\infty}\frac{j\pi\omega\delta(\omega)+1}{m(\omega)+\frac{c(\omega)}{j\omega}}=\frac{1}{m(\infty)} \tag{13}$$

Therefore $a(0^+)=1/m(\infty)$ where $m(\infty)$ is the apparent mass at ω=infinity.

Equating $t=0$ at eq. (11), we get

$$\frac{1}{m(\infty)}-\frac{1}{m(0)}=\frac{2}{\pi}\int_0^\infty \frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}d\omega \tag{14}$$

Now, let us calculate numerically the 2nd term of eq. (11) for the Japanese first atomic ship at model size ($L_{pp}=2.0\,m$, scale=1/58). Added mass $m_0(\omega)$ and damping $c(\omega)$ are calculated by strip method making use of Tasai's value of added mass and damping for each section. Results are as shown in Fig. 1. The integrand of eq. (11) is also shown in Fig. 1 where we know that the integrand is positive function. In vertue of the positive function, we may use an approximation at inverse transformation of the integrand (See Appendix). Result is as follow

$$\begin{aligned}
&\frac{2}{\pi}\int_0^\infty \frac{c(\omega)}{m^2(\omega)\omega_1{}^2+c^2(\omega)}\cos\omega t d\omega \\
&=\frac{2}{\pi t^2}\left[\frac{R_0}{\omega_2-\omega_1}(\cos\omega_2 t-\cos\omega_1 t)\right. \\
&+\frac{R_4}{\omega_8-\omega_3}(\cos\omega_3 t-\cos\omega_8 t) \\
&+\frac{R_3-R_4}{\omega_7-\omega_3}(\cos\omega_3 t-\cos\omega_7 t) \\
&+\frac{R_2-R_3}{\omega_6-\omega_3}(\cos\omega_3 t-\cos\omega_6 t) \\
&+\frac{R_1-R_2}{\omega_5-\omega_3}(\cos\omega_3 t-\cos\omega_5 t) \\
&\left.+\frac{R_0-R_1}{\omega_4-\omega_3}(\cos\omega_3 t-\cos\omega_4 t)\right]
\end{aligned} \tag{16}$$

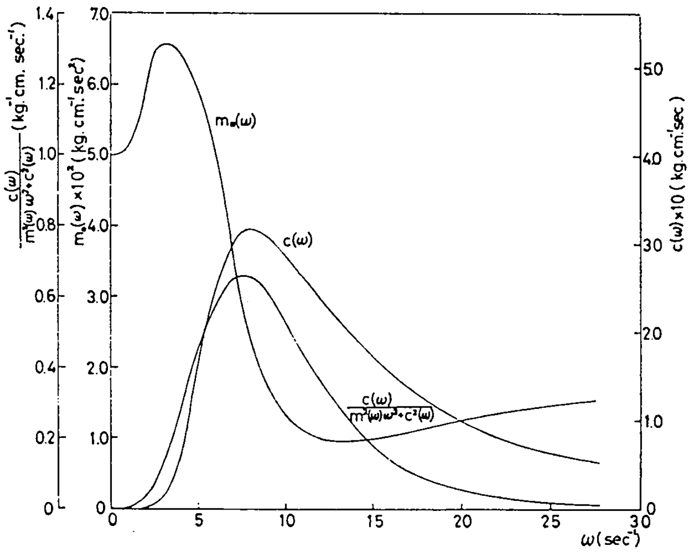

Fig. 1 Added mass $m_0(\omega)$ and damping coefficient $c(\omega)$ of the first atomic Ship of Japan at model scale

Thus, the equivalent added mass $\bar{m}_v''$ as defined in the Introduction will be obtained as follows

$$m^*+\bar{m}_v''=\frac{f(t)}{a(t)}=\frac{1}{a(t)} \quad (t>0) \tag{17}$$

where m^* is the mass of the ship itself.

It should be noted that the initial value of $\bar{m}_v''$ is equal to $m_0(\infty)$. In a same way, we can define an equivalent added mass $\bar{m}_v'$ which will be given in the following formula

$$m^*+\bar{m}_v'=\frac{\int_0^t f(\tau)d\tau}{v(t)}=\frac{t}{v(t)} \tag{18}$$

where $v(t)$ is the terminal velocity.

Since

$$v(t)=\frac{t}{m(0)}+\frac{2}{\pi}\int_0^\infty \frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cdot\frac{\sin\omega t}{\omega}d\omega \tag{19}$$

$\bar{m}_v'$ is obtained as follows

$$m^*+\bar{m}_v'=\frac{1}{\dfrac{1}{m(0)}+\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cdot\frac{\sin\omega t}{\omega t}d\omega} \tag{20}$$

If we put $t\to0$ in eq. (20) and refering eq. (14), we know that the initial value of $\bar{m}_v'$ is equal to $m_0(\infty)$. Another equivalent added mass which gives exact value of absorbed energy can be defined as follows

$$m^*+\bar{m}_v=\frac{\int_0^t f(\tau)v(\tau)d\tau}{\frac{1}{2}v^2(t)}=\frac{\int_0^t v(\tau)d\tau}{\frac{1}{2}v^2(t)} \tag{21}$$

The initial value of $\bar{m}_v$ is also $m_0(\infty)$.

b) In case of a ramp input

Next, let us consider a ramp input as shown in eq. (23)

$$f(t)=tU(t)=\begin{cases}0, & t<0\\ t, & t\geqq 0\end{cases} \tag{23}$$

The Fourier transform of $f(t)$ is

$$F(w)=j\pi\frac{d\delta(\omega)}{d\omega}-\frac{1}{\omega^2} \tag{24}$$

Since $R(\omega)$ and $I(\omega)$ of eq. (5) are

$$R(\omega)=\frac{-m(\omega)-c(\omega)\omega\pi\delta'(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}$$

$$I(\omega)=\frac{-c(\omega)+m(\omega)\omega^3\pi\delta'(\omega)}{m^2(\omega)\omega^3+c^2(\omega)\cdot\omega} \tag{25}$$

We get $a(t)$ as follows

$$a(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{c(\omega)}{m^2(\omega)\omega^3+c^2(\omega)\omega}\sin\omega t d\omega$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\omega)\omega^2\pi\delta'(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\sin\omega t d\omega \tag{26}$$

Refering that; $\omega\delta'(\omega)=-\delta(\omega)$ (27)

the 2nd term will be deformed as follows

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(\omega)\omega^2\pi\delta'(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\sin\omega t d\omega$$

$$=\int_{-\infty}^{\infty}\frac{m(\omega)\omega\delta(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\sin\omega t d\omega=\frac{t}{m(0)}$$

Since the integrand of the 1st term of eq. (26) is even function, we get

$$a(t)=\frac{t}{m(0)}+\frac{2}{\pi}\int_{0}^{\infty}\frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cdot\frac{\sin\omega t}{\omega}d\omega \tag{29}$$

We know that $a(t)$ in eq. (29) is exactly same as $v(t)$ of eq. (19). This is the matter of course because a ramp input $tU(t)$ is an integal form of a step input $U(t)$[6]. If we know the response $s(t)$ to a step input, response $g(t)$ to an arbitrary input $f(t)$ is given as follows.

$$g(t)=|H(0)|\cdot f(-\infty)+\int_{-\infty}^{\infty}f'(\tau)\tau(t-\tau)d\tau \tag{30}$$

Where $|H(0)|$ is a value of the Fourier spectrum of a linear system function at frequency $\omega=0$

For the ramp input,

$$f(-\infty)=0$$

$$f'(\tau)=\begin{cases}0, & \tau<0\\ 1, & \tau\geqq 0\end{cases} \tag{31}$$

Since $s(t-\tau)=0$ for $t<\tau$, range of integral becomes $-\infty$ to t. Finally we get,

$$g(t)=\int_{-\infty}^{t}f'(\tau)s(t-\tau)d\tau=\int_{0}^{t}s(t-\tau)d\tau$$

$$=\int_{0}^{t}s(\tau)d\tau \tag{32}$$

If we put $s(t)=a(t)$ in eq. (11), $g(t)$ should be the terminal acceleration for a ramp input. Therefore, $\bar{m}_y''$ will be given by the following formula.

$$m^*+\bar{m}_y''=\frac{1}{\frac{1}{m(0)}+\frac{2}{\pi}\int_{0}^{\infty}\frac{c(\omega)}{m^2(\omega)\omega^2+c^2(\omega)}\cdot\frac{\sin\omega t}{\omega t}d\omega} \tag{33}$$

Since the right hand side of eq. (33) is exactly equal to that of eq. (20), we know that $\bar{m}_y'$ for a step input is the same as $\bar{m}_y''$ for a ramp input. $\bar{m}_y'$ and $\bar{m}_y$ for a ramp input can be obtained in the same manner.

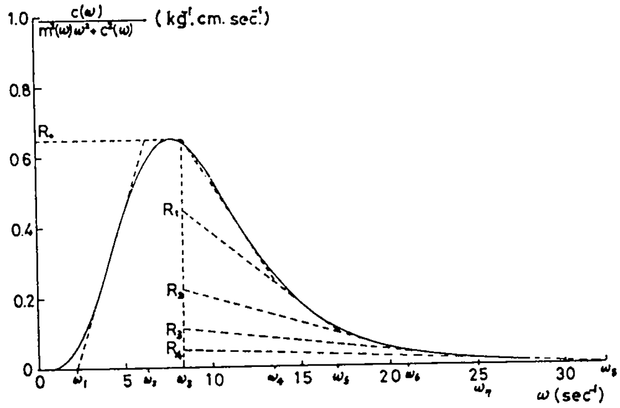

Fig. 2 Geometrical approximation of the function $c(\omega)/m^2(\omega)\omega^2+c^2(\omega)$

The equivalent added masses $\bar{m}_y''$, $\bar{m}_y'$, $\bar{m}_y$ thus obtained are functions of time. Since the initial values for these three kinds of added masses are all equal to $m_0(\infty)$, if the duration of a collision is very short, then $\bar{m}_y''$, $\bar{m}_y'$, and $\bar{m}_y$ are practically equal to $m^0(\infty)$. To examine the dependancy of the equivalent added masses to the duration, the equivalent added mass coefficients $\bar{m}_y''/m^*$, $\bar{m}_y'/m^*$, and $\bar{m}/m^*$ are calculated of the 1st atomic ship as shown in Fig. 3, 4, and 5. Abscissa of these figures is the duration and

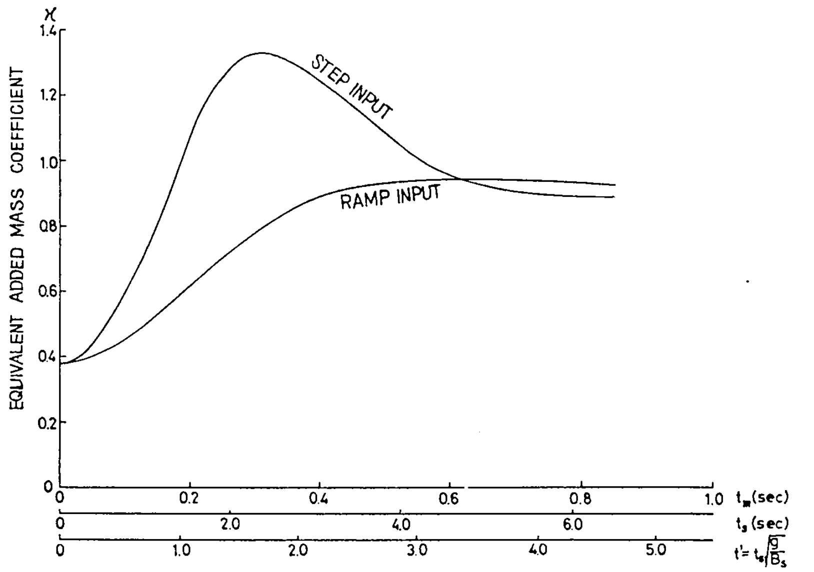

Fig. 3 Equivalent added mass $\overline{m}_y''/m^*(=\kappa)$

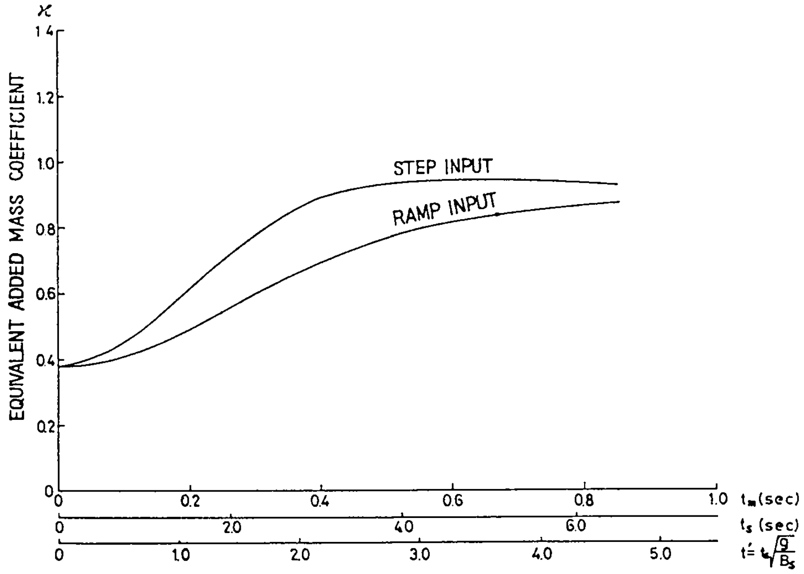

Fig. 4 Equivalent added mass $\overline{m}_y'/m^*(=\kappa)$

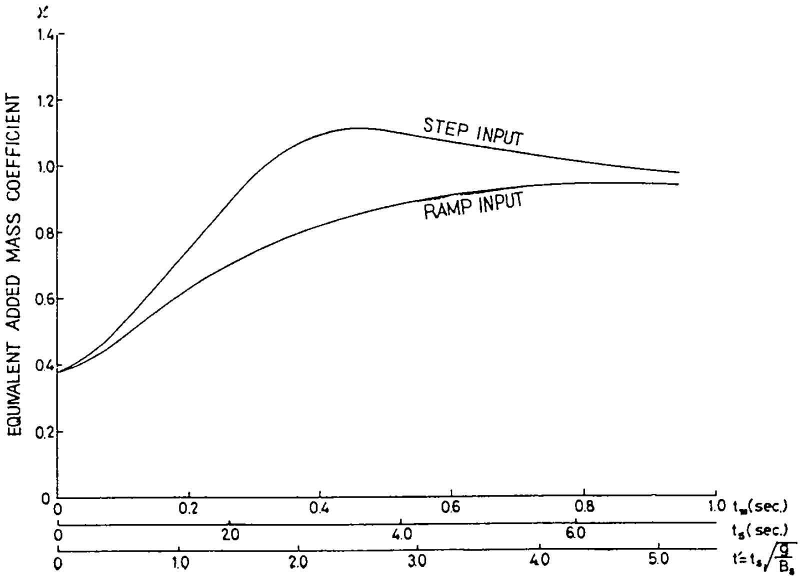

Fig. 5 Equivalent added mass $\overline{m}_y/m^*(=\kappa)$

scale for model time t_m, full scale time t_s, and a non-dimensionalized time t' are shown as the reference.

From these figures, we can notice that all kinds of the equivalent added masses increase as the duration increases. In case of the step input, the aforementioned trend is most remarkable. For any kind of input, the following relation is valid.

$$\bar{m}_y'' > \bar{m}_y > \bar{m}_y'$$

It should be also noted that the equivalent added mass changes its value in accordance with type of the external force.

3. Experiments

To examine the theoretical value of the equivalent added mass, the authors conducted experiments as described in the following section.

A 2.0 m model of the 1st atomic ship is connected to a frictionless guide so that it is only allowed to sway (see Fig. 6). A horizontal type accelerometer was set on the model. A nylon string, connected to the model at its center of gravity, is lead horizontally and connected to a weight through pulley so that the model is pulled horizontally by the weight. Another string is stretched in opposite side and is fixed to restrain the model to sway. The principal dimensions of the model are shown in Table 1. When the fixed nylon string is cut, the model would

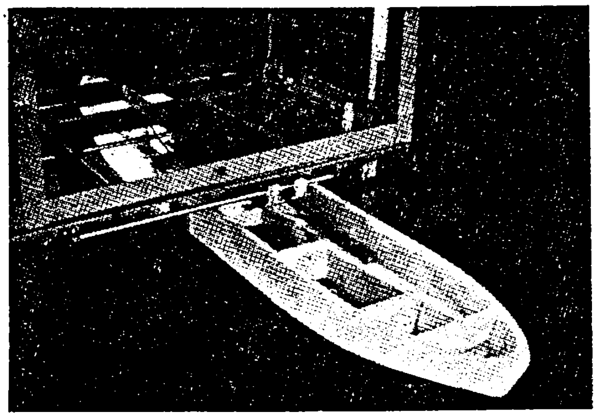

Fig. 6 Frictional guide for measurement of lateral acceleration

Table 1 Principal dimensions of the model used at the experiment

L_{pp}	2,000.0 mm
B	327.6 mm
D	227.6 mm
d	119.0 mm
Δ	53.0 kg
C_b	0.664
C_p	0.672
C_w	0.826
$C_{⊗}$	0.988
C_{vp}	0.804
⊗B	39.6 mm *aft*

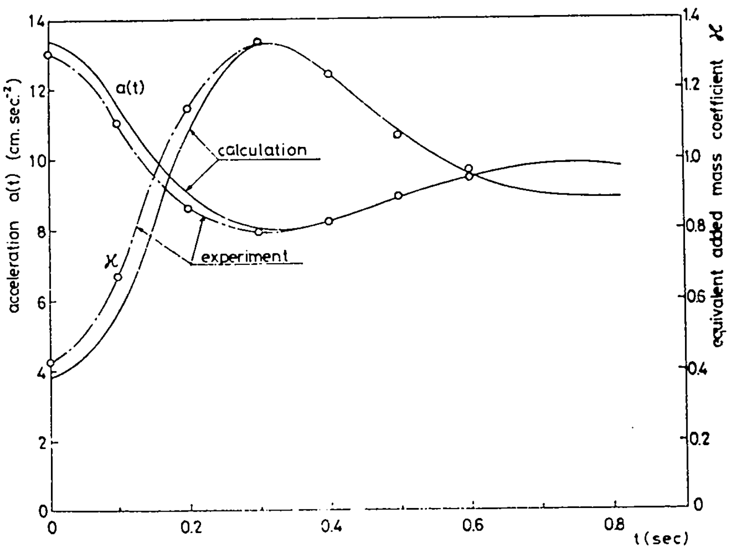

Fig. 7 Acceleration $a(t)$ and equivalent added mass $m_y''/m^*(=\kappa)$

be acted by a stepwise external force and be accelerated to sway. The acceleration is recorded by an oscillograph. An example of record with 1 kg. weight is shown in Fig. 7.

The acceleration thus induced by the stepwise force by the weight is recorded on an oscillograph. Measured acceleration in case of 1 kg weight is as shown in Fig. 7 on time basis. The equivalent added mass coefficient χ $(=\bar{m}_y''/m^*)$ obtained by measured acceleration is also shown in Fig. 7 for different duration. Calculated values of the acceleration and the equivalent added mass coefficent are also shown in Fig. 7 by solid lines. There are slite difference between measured and calculated values at initial stage, but it can be said that they showed reasonable agreement. Speed of the ship $v(t)$, as well as the translation $s(t)$ of the ship are also obtained by integrating acceleration and the equivalent added mass coefficient based on the speed $(\bar{m}_y'/m^*)$ and based on the absorbed energy $(\bar{m}_y/m^*)$ are also obtained as shown in Figs. 8 and 9. Agreement with calculated values were also good.

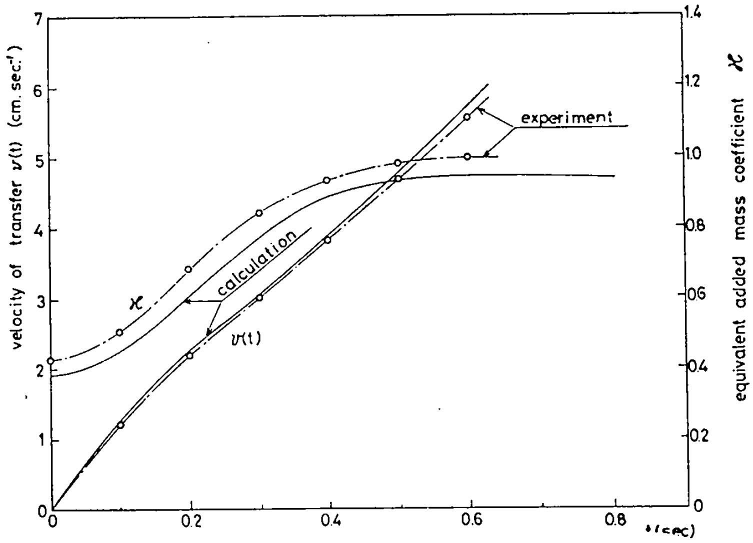

Fig. 8 Velocity $v(t)$ and equivalent added mass $\bar{m}_y'/m^*(=\kappa)$

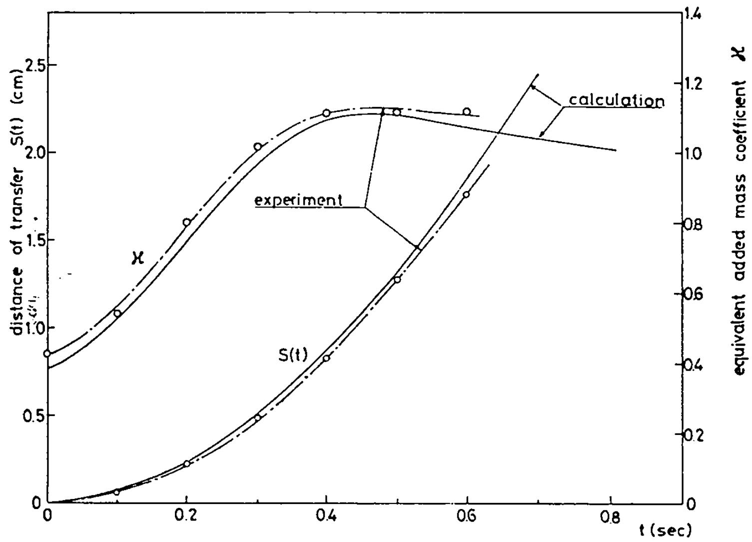

Fig. 9 Translation $s(t)$ and equivalent added mass $\bar{m}_y/m^*(=\kappa)$

4. Estimation of the duration of collisions

Since the equivalent added mass increases as the duration of a collision.

a) In case of step input

Let us assume the external force is a stepwise one of magnitude f_0. Denoting the duration τ, the equivalent added masses $\tilde{m}_x$, $\tilde{m}_y$ and expressing the striken ship by sub-suffix A and the striking ship by sub-suffix B, we can write the equation of motions of transverse motion of the A ship and longitudinal motion of the B ship as follows:

$$\left.\begin{aligned}(m_B+\tilde{m}_{Bx})\frac{du_B}{dt}&=-f_0\\(m_A+\tilde{m}_{Ay})\frac{dv_A}{dt}&=f_0\end{aligned}\right\} \tag{34}$$

On the other hand, denoting the penetration of the B ship into the A ship as S we get

$$S=\int_0^\tau(u_B-v_A)dt=U_{B0}\tau-\frac{f_0\tau^2}{2}\left(\frac{1}{m_B+\tilde{m}_{Bx}}+\frac{1}{m_A+\tilde{m}_{Ay}}\right) \tag{35}$$

where U_{B0} is the initial speed of the B ship. Then, if we denote the terminal velocity of both ships as U (speed at $t=\tau$), U can be written as follows

$$\frac{f_0\tau}{m_B+\tilde{m}_{Bx}}=U_{B0}-U,\quad \frac{f_0\tau}{m_A+\tilde{m}_{Ay}}=U \tag{36}$$

Therefore substituting (36) into (35), we get

$$S=\frac{U_{B0}\tau}{2},\qquad \tau=\frac{2S}{U_{B0}} \tag{37}$$

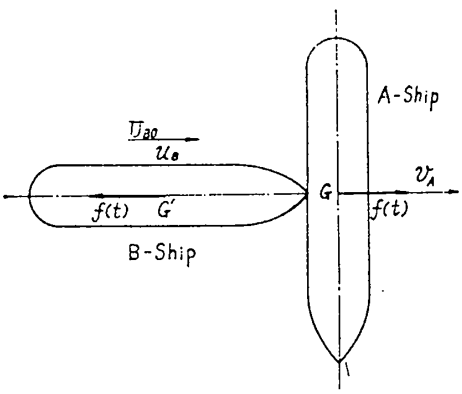

Fig. 10 Lateral Collision

(37) shows that "the duration is a function of only the penetration and the initial speed of the striking ship. It is proportional to the penetration, and inversely proportional to the initial speed". If a comparatively slow ship deeply penetrates a soft structured ship, the duration should be appreciable.

b) In case of a ramp wise force.

Replacing f_0 in (34) by $f(t)=t$, we get

$$S=\int_0^\tau(u_B-v_A)dt=U_{B0}\tau-\frac{\tau^3}{6}\left(\frac{1}{m_B+\tilde{m}_{Bx}}+\frac{1}{m_A+\tilde{m}_{Ay}}\right)=\frac{2}{3}U_{B0}\tau,\quad \tau=\frac{3S}{2U_{B0}} \tag{38}$$

Comparing (38) and (37), we know that for the same S and U_{B0}, the duration of case b) is 3/4 times the duration of case a).

For instance, assuming the penetration to be 4 m, the initial speed to be 18 kts, the duration of stepwise input is 0.865 sec. while the duration of ramp input is 0.649 sec. The equivalent added mass coefficient based on the absorbed energy for both cases are 0.57 and 0.47 respectively which are fairly greater then $m_0(\infty)/m^*(=0.382)$. Values of equivalent added mass corresponding to various combinations of the penetration and the initial speed are shown in Table 2 for stepwise and ramp wise input.

5. Conclusion

(1) Minorsky has used 0.4 for the added mass coefficient of a striken ship in calculating the absorbed energy by a collision. In this paper, the authors showed that there is no time-independent constant value of added mass coefficient. However, it is possible to introduce an equivalent added mass coefficient which is a constant and will give exact value of terminal value of acceleration when it is used as the added mass coefficient in the equation of motion. In the same senes, an equivalent added mass coefficient based on terminal velocity of the striken ship and an equivalent added mass coefficient based on the absorbed energy will be defined.

Table 2 Duration of collision τ and equivalent added mass coefficient χ of the first atomic ship (at actual ship)

Initial speed \ Penetration / Input		2 m Step	2 m Ramp	4 m Step	4 m Ramp	6 m Step	6 m Ramp	8 m Step	8 m Ramp	10 m Step	10 m Ramp
6	τ (sec)	1.30	0.975	2.59	1.94	3.89	2.92	5.18	3.89	6.48	4.86
	χ	0.562	0.424	0.830	0.553	0.935	0.683	0.934	0.700	0.930	0.828
10	τ (sec)	0.777	0.583	1.56	1.17	2.34	1.76	3.12	2.34	3.89	2.92
	χ	0.460	0.392	0.623	0.448	0.787	0.526	0.905	0.610	0.935	0.683
14	τ (sec)	0.557	0.418	1.11	0.833	1.67	1.25	2.23	1.67	2.79	2.09
	χ	0.504	0.388	0.522	0.410	0.650	0.459	0.763	0.514	0.864	0.575
18	τ (sec)	0.432	0.324	0.865	0.649	1.30	0.975	1.73	1.30	2.16	1.62
	χ	0.415	0.385	0.475	0.396	0.562	0.424	0.663	0.465	0.751	0.507
22	τ (sec)	0.354	0.266	0.708	0.531	1.06	0.795	1.38	1.04	1.77	1.33
	χ	0.409	0.384	0.449	0.390	0.511	0.405	0.583	0.431	0.670	0.468
26	τ (sec)	0.299	0.225	0.598	0.446	0.898	0.674	1.20	0.900	1.50	1.13
	χ	0.403	0.384	0.435	0.389	0.480	0.399	0.543	0.416	0.608	0.440

(2) The equivalent added masses thus defined are functions of the duration of a collision, and the longer the duration the greater the equivalent added mass coefficient. When the duration is zero, all kinds of the equivalent added mass coefficients coincide to $m_y(\infty)$ which was used by Minorsky.

(3) The acceleration, speed, and the equivalent added mass coefficients which are calculated by superposition of linear equation of motion based on the frequency agreed quite well with experimental results. This fact shows that linear superposition of equation of motion which has frequency dependent coefficients is valid for this kind of problems.

Appendix

Some approximate methods of calculating the eq. (16) were already described in a few published reference books[6),7)] but for convenience we shall dare to describe the methods in this appendix. Let us consider the inverse Fourier transform of $R_1(\omega)$ of which the form is trapezoid as shown in Fig. A-1.

$$r_1(t)=\frac{2}{\pi}\int_0^{\infty} R_1(\omega)\cos\omega t d\omega \qquad \text{(A-1)}$$

The function $R_1''(\omega)$ which is obtained by differentiating $R_1(\omega)$ with respect to frequency ω, has four impulse functions as shown in Fig. A-2.

$$R_1''(\omega)=\frac{R}{\omega_2-\omega_1}\{\delta(\omega-\omega_1)-\delta(\omega-\omega_2)\}$$
$$-\frac{R}{\omega_4-\omega_3}\{\delta(\omega-\omega_3)-\delta(\omega-\omega_4)\} \qquad \text{(A-2)}$$

By the way, if we denote the inverse Fourier transform of $R_1(\omega)$ as $r_1(t)$, $-t^2r_1(t)$ stands for the inverse Fourier transform of $R_2''(\omega)$. Namely,

$$-t^2r_1(t)=\frac{2}{\pi}\int_0^{\infty} R_1''(\omega)\cos\omega t\, d\omega \qquad \text{(A-3)}$$

Substituting the eq. (A-2) into this equation and using the next relationship,

$$\int_0^{\infty}\delta(\omega-\omega_1)\cos\omega t d\omega=\cos\omega_1 t \qquad \text{(A-4)}$$

we may obtain the inverse Fourier transform $r_1(t)$ as follows.

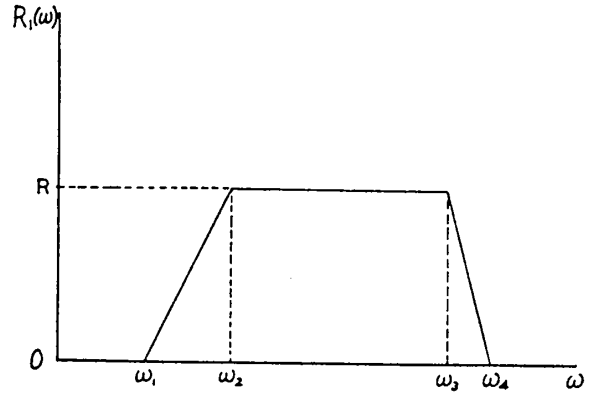

Fig. A-1

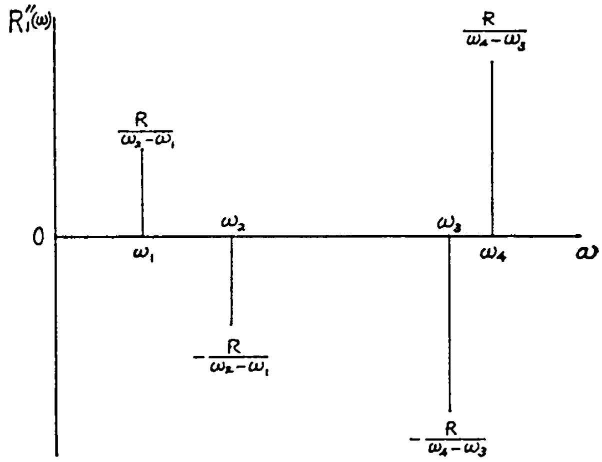

Fig. A-2

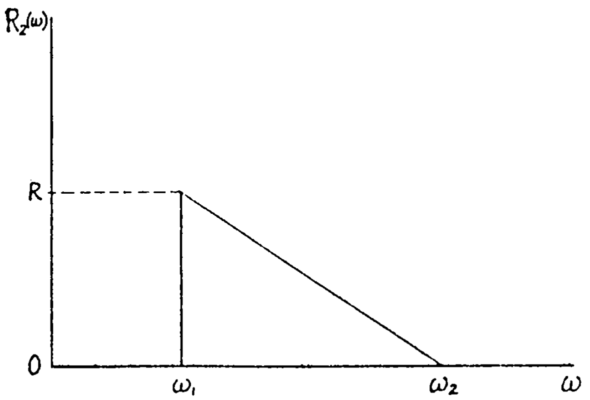

Fig. A-3

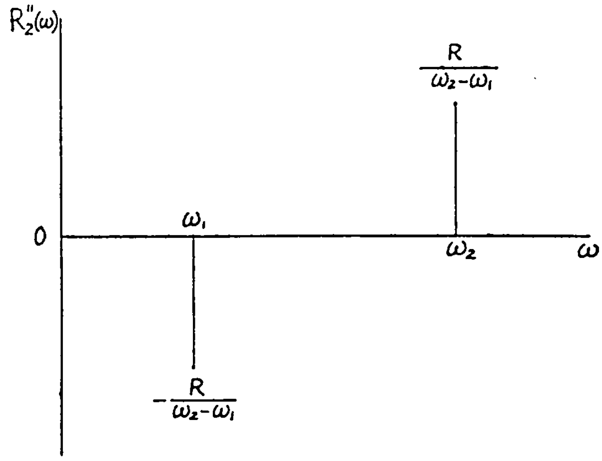

Fig. A-4

$$r_1(t) = -\frac{2}{\pi t^2}\left\{\frac{R}{\omega_2-\omega_1}(\cos\omega_1 t - \cos\omega_2 t) - \frac{R}{\omega_4-\omega_3}(\cos\omega_3 t - \cos\omega_4 t)\right\} \tag{A-5}$$

Similarly in case of $R_2(\omega)$, of which the form is triangle as shown in Fig. A-3, we can obtain the inverse transform $r_2(t)$.

$$r_2(t) = \frac{2}{\pi}\int_0^\infty R_2(\omega)\cos\omega t d\omega \tag{A-6}$$

The function $R_2''(\omega)$ has two impulse functions

$$R_2''(\omega) = -\frac{R}{\omega_2-\omega_1}\{\delta(\omega-\omega_1)-\delta(\omega-\omega_2)\} \tag{A-7}$$

$$r_2(t) = \frac{2}{\pi t^2}\cdot\frac{R}{\omega_2-\omega_1}(\cos\omega_1 t - \cos\omega_2 t) \tag{A-8}$$

Since the function $c(\omega)/(m^2(\omega)\omega^2+c^2(\omega))$ is composed of two trapezoids and five triangles, we can obtain the desired inverse transform (16) by adding the above stated inverse transforms.

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