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9. On the Modified Zigzag Maneuver and its Application

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Summary

In case of less stable ships or unstable ships such as full bodied ship, there are difficulties in conducting ordinary zigzag maneuver at a small rudder angle, say 5 degrees. In order to examine the course keeping qualities of such ships, the authors proposed a modified zigzag maneuver, which was distinguished from ordinary zigzag maneuver in the respect that the rudder angle was not equal to the switching heading angle. As an example, results of ordinary and modified zigzag tests on two full supertankers are reported and the maneuverability indices, T^* and K^*, obtained from them are compared with each other. In the latter half of this paper, it is described that modified zigzag maneuver is valid to obtain frequency response function of maneuverability.

Introduction

In case ships with stable couse keeping quality, the zigzag maneuver proposed by Kempf[1] can be executed with ease even at small rudder angle. It has been a useful method to searth the maneuverability of ships by making use of the first order system analysis[2][3]. However, this method is not suitable for determining the maneuverability of less stable or unstable ships, because the trajectory of such ships becomes remarkable asymmetric or diverges in spite of the compensatory steering so that the zigzag manever is unable to be continued. For the sake of removing the difficulty, the authors have proposed to execute a different type of zigzag maneuver (hereafter it will be called as the modified zigzag maneuver) which is distinguished from the normal zigzag maneuver in the repect that the rudder angle δ^* is not always equal to the switching course angle ψ^* at which the rudder is reversed[4].

In this paper, the authors shall again introduce the tenor of the article 4) with some examples together, and moreover describe the application of the modified zigzag maneuver to determination of the frequency response function of steering.

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1. Modified Zigzag Maneuver

1.1 Difficulties in executing the normal zigzag maneuver in case of less stable or unstable ships

As stated in the introduction, there are some difficulties in executing the normal zigzag maneuver at a small rudder angle, to say 5 degrees, in case of less stable or unstable ships such as full super tankers, because the trajectory of ship in process of the zigzag maneuver diverges to infinity or if it were not so, the difference of the course angle from the original course grows so large that the zigzag maneuver is unable to be continued. Fig. 1 shows the qualitative relationship between the rudder angle δ^* used at the zigzag maneuver and the amplitude $\bar{\psi}$ of course angle as well as the period τ' (=distance run/ship's length) of limit cycle in both cases of stable and unstable ships. As shown in fig. 1, it is impossible at all to execute the normal zigzag maneuver at small rudder angle in case of unstable ships, and even in case of less stable ships it is almost impossible from the

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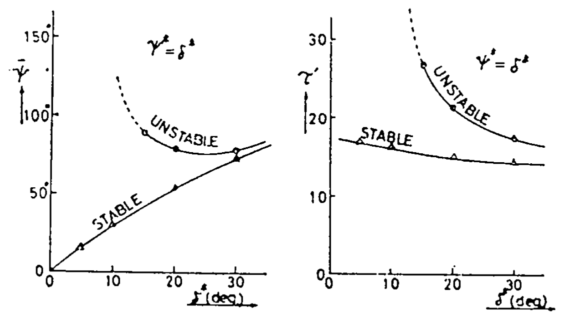

Fig. 1 Amplitude of head angle $\bar{\psi}$ and period τ' $(=\tau U/L)$

practical point of view.

1.2 Proposition of modified zigzag maneuver

In spite of the difficulties in executing the normal zigzag maneuver, even less stable or unstalbe ships can be steered on a straight course without difficulty. The reason will be that at the course keeping maneuver the rudder is reversed to the compensatory direction before the deviation of course angle grows larger than 1 degree even in case of manual steering; much less the deviation is in case of automatic steering. Therefore, the angular velocity as well as the drifting velocity do not develope to create unstable hydrodynamic moment. Hence, the ships can quickly response to even small rudder angle. As an example, the results of zigzag maneuver tests on the prescribed unstable ship whose equation of motion is reprsented by a second order nonlinear equation (11), are shown fig. 2 (a)–(f) at which the rudder angle is kept 5 degrees while the switching course angle is varied from 5° to 0.2°. In this example, the change of course angle grows gradually larger with the lapse of time in case of 1 degree switching course angle, while it reaches to a steady state, that is limit cycle, in case of that less than 0.8°. The ship motion at the modified zigzag maneuver with small switching course angle seems to be more similar to the actual ship motion at the course keeping maneuver than the normal zigzag maneuver. Therefore, the authors have proposed that it is more

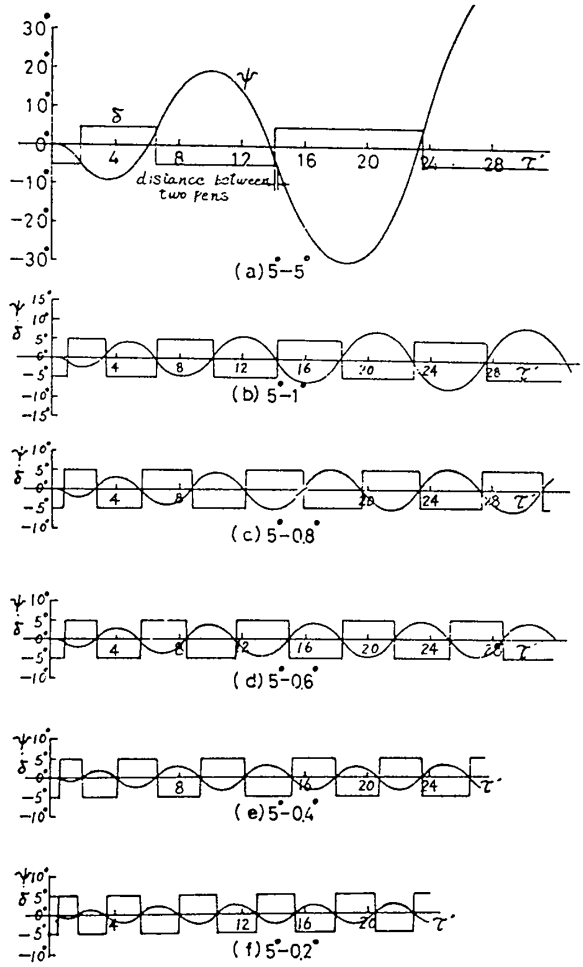

Fig. 2 Simulation of zigzag maneuver in case of using non-linear equation (11) ($T_1'=-17.75$, $T_2'=0.485$, $T_3'=0.895$, K' -9.15, $p'=-8.6$)

suitable to execute the modified zigzag maneuver in order to examine the course keeping qualities of ships.

In the following paragraphs, we shall compare the course keeping qualities obtained by the first order system analysis of the normal and the modified zigzag maneuvers with each other.

1.3 The aim of the first order system analysis

Comparing with the results obtained from the first order system analysis of 15°–15° normal, 5°–5° normal and 5°–1° modified zigzag maneuvers (the former of the numbers connected by the symbol—means the

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amount of the rudder angle used at zigzag maneuvers and the latter does the switching course angle), we can draw the inference that K, T indices from 5°-5° normal zigzag maneuver are remarkable larger than those from 15°-15° normal one, while K, T indices from 5°-1° modified zigzag maneuver are smaller than those from 15°-15° normal and 5°-5° normal zigzag maneuvers. This seems to coincide exactly with the fact that the responses of less stable or unstable ships to such modified zigzag maneuver as 5°-1° modified one, is virtually similar to those of stable ships, and therefore even unstable ships such as full super tankers are able to be steered without so much difficulties.

Then, we shall examine the aim of the first order system analysis in order to decide which K, T indices are well representative of the course keeping quality. For the sake of brevity, we assume that the equation of motion may be described by the second order linear equation as follows.

$$T_1T_2\frac{d^3\psi}{dt^3}+(T_1+T_2)\frac{d^2\psi}{dt^2}+\frac{d\psi}{dt}=K\delta+KT_3\frac{d\delta}{dt} \qquad (1)$$

where the symbols used here follow the article 2). The first order approximation of this equation proposed by Nomoto[2)] is

$$T\frac{d^2\psi}{dt^2}+\frac{d\psi}{dt}=K\delta$$

$$T=T_1+T_2-T_3 \qquad (2)$$

The solution $\psi(t)$ of eq. (2) will agree exactly with that of eq. (1) after the lapse of infinitely long time, while at the initial stage

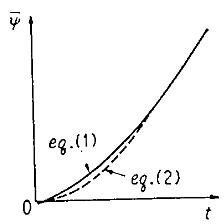

Fig. 3 Comparison between the response of first order system to stepwise steering and that of second order system

the former solution is smaller than the latter one as shown by the dotted line in fig. 3. The less stable a ship is, the larger the difference between the solutions of eqs. (1) and (2) is.

Now we assume that there exist two ships, of which the course keeping qualities are represented by eqs. (1) and (2) respectively. Comparing with the records of course angle and rudder angle at the zigzag maneuver, such differences as shown in fig. 4 by the solid and dotted lines may be found between

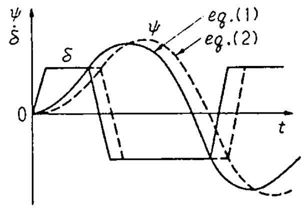

Fig. 4 Comparison between zigzag maneuver of first order system and that of second order system

these two ships. Namely, in case of ships whose equation of motion is described by eq. (1), the timing of reversing the rudder angle advances and the overshooting course angle decreases compared with those of ships of eq. (2). Therefore, the K and T indices of ships described by eq. (1) are smaller than those of ship by eq. (2), and hence the former ships seem to be of more stable course keeping quality than the latter ship.

This tendency will be much exaggirated as the switching course angle decreases. In order to distinguish the K, T indices obtained from the first order system analysis of the zigzag maneuver of ships described by eq. (1) from those of eq. (2), let's use the symbols K^* and T^*. These K^* and T^* values of stable ship obtained from the analogue simulation of various kinds of modified zigzag maneuvers are shown in figs. 5 and 6. The prescribed values of T_1', T_2', T_3' and K' used at these simulation are 12.68, 0.420, 0.893 and 5.82 respectively. The K and T indices of the eq. (2) correspond exactly to the K^* and T^* indices obtained from the modified zigzag

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maneuvers of infinitesimal rudder angle. The larger the rudder angle of the zigzag maneuver is, the smaller the K^* and T^* values are. This tendency is remarkable in case of small ψ^*/δ^* ratio.

The K^* and T^* indices obtained from the various kinds of zigzag maneuvers are independent of the amount of rudder angle δ^* and are constant, only if the ratio ψ^*/δ^* remains unchanged. But the K^* and T^* values do not coincide with the K and T values of eq. (2). This difference between the K^*, T^* values and the K, T values increases as the course keeping quality becomes worse. The K, T values of unstable ships are always negative, but there may exist the positive K^* and T^* values in case of certain small ψ^*/δ^* ratio. Therefore such modified zigzag maneuvers are able to be executed even in case of unstable ships.

1.4 What is the purpose of the zigzag maneuver?

As stated above, the K^* and T^* values determined from the modified or normal zigzag maneuvers are usually different from the K and T indices of the first order system analysis. Then, which indices are best representative of the maneuvering characteristics of ships?

Principal modes of steering motion of ships will be classified in the following items, which

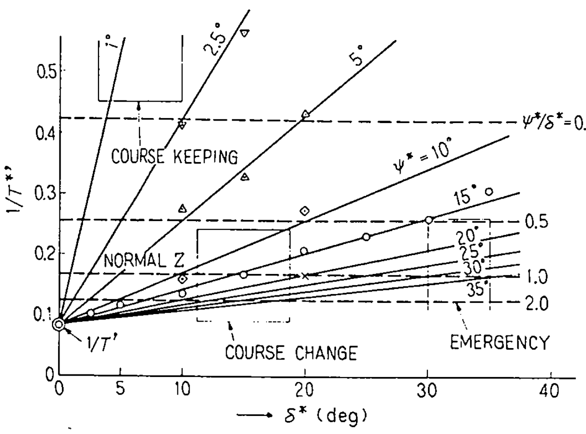

Fig. 5 T^* of a stable ship obtained from various kind of modified zigzag maneuvers

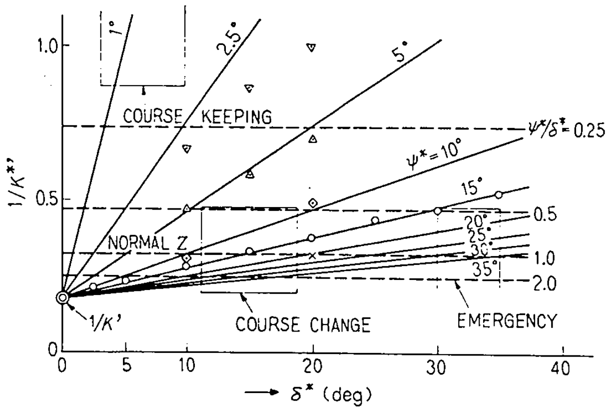

Fig. 6 K^* of a stable ship obtained from various kind of modified zigzag maneuvers

are shown in figs. 5 and 6 qualitatively.

a) course keeping maneuver
b) course change maneuver
c) emergency maneuver

Among these items, the b) and c) are well covered by normal zigzag maneuvers of $\delta^*=15°$ and 35° respectively, because in these modes of steering motion the rudder angles of 15° or 35° are frequently used and kept unchanged until the changes of course angle grow to the similar extent as the amounts of the used rudder angle.

Accordingly, the K^* and T^* values thus obtained from such normal zigzag maneuvers, to say 15°–15° and 35°–35° normal ones, will be well representative of the maneuvering characteristics of the items b) and c). On the other hand, in the mode a) the helmsman endeavors to keep the change of course angle within less than 1 degree by steering the rudder of more or less than 5 degrees. Therefore, the modified zigzag maneuvers of small ψ^*/δ^* ratios, for instance 5°–1° modified one, are more adequate and more practical than the normal zigzag maneuver to examine the course keeping quality of ships. The K^* and T^* values obtained from these modified zigzag maneuvers, as stated already, are smaller than those from the normal zigzag maneuvers. In other words, the former K^* and T^* values suggest more stable course keeping quality than the latter ones do.

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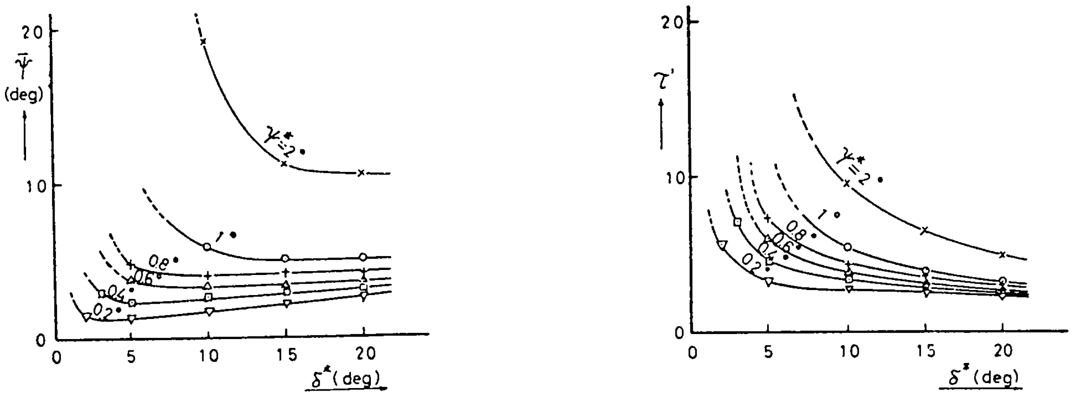

a) b)

Fig. 7 Amplitude of head angle $\bar{\psi}$ and period τ' $(=\tau U/L)$ of limit cycles at the various kind of modified zigzag maneuvers carried out with an unstable ship ($T_1'=-17.75$, $T_2'=0.485$, $T_3'=0.895$, $K'=-9.15$, $p'=-8.6$)

1.5 Conditions for the existance of stable limit cycles.

Even in case of unstable ships, whose normal zigzag maneuvers can not be executed because of divergence of their course angle, if we choose an adequate small switching course angle ψ^* compared with rudder angle δ^*, the modified zigzag maneuver will be able to be executed, so that on the phase portrait the trajectory of steering motion will reach a certain stable limit cycle at last. In figs. 7 (a) and (b), as an example, the amplitudes of course angle $\bar{\psi}$ and dimensionless periods τ' of such stable limit cycles are shown with respect to some combinations of rudder angle δ^* against the switching course angle ψ^* in case of an unstable ship whose equation of motion is nonlinear. Hereafter for the present, we shall concentrate our attention to only such cases as the equation of motion is linear. From the studies stated above, it may be deduced that the zigzag maneuvers with large rudder angle δ^* in comparison with the switching course angle ψ^* are able to be executed, while in the reverse case, to say small δ^* compared with ψ^*, they are unrealizable. Therefore, it will be suspected that there exists a certain critical value of ψ^*/δ^* beyond which the zigzag maneuver of unstable ships can not reach the stable limit cycle. The amount of such critical value of the ψ^*/δ^* ratio will be considered as one kind of criterion for judging the course keeping quality. Then in this paragraph, we shall investigate the conditions under which the stable limit cycle is able to be reached by interminable execution of zigzag maneuver. In fig. 8 is shown the block diagram of modified zigzag maneuver, where the symbols

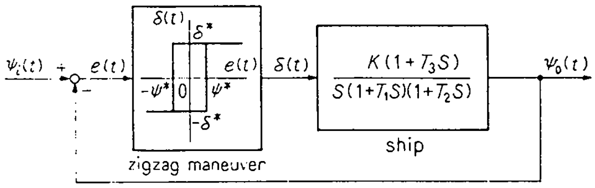

Fig. 8 Block diagram of zigzag maneuver

ψ^* and δ^* are representative of the switching course angle and rudder angle respectively. Besides, $\psi_i(t)$, $\psi_0(t)$, $e(t)$ and $\delta(t)$ mean some time-dependent function, to say the desired course angle, actual course angle, error of course angle and rudder angle respectively. However, the desired course angle $\psi_i(t)$ may be made null without lost of generality.

In case of stable ships, the rudder angle $\delta(t)$ and course angle $\psi_0(t)$ (hereafter will be written as $\psi(t)$ for the aim of brevity) will reach the steady oscillatory state, to say limit cycle, after interminable execution of zigzag steering. The existance of such limit cycles can be assured by the following re-

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asoning. The frequency characteristics of the rudder angle $\delta(t)$ which varies rectangularly against the time, can be well described by the so-called describing function K_{eq} as follows:[5)]

$$\left.\begin{aligned} K_{eq}&=g(\bar{\psi})+jb(\bar{\psi})=\frac{4\delta^*}{\pi\bar{\psi}}\cos\psi_1-j\frac{4\delta^*}{\pi\bar{\psi}}\sin\psi_1 \\ \psi_1&=\sin^{-1}(\psi^*/\bar{\psi}) \end{aligned}\right\} \quad (3)$$

Assuming that the error signal $e(t)$ is sinusoidal function of time, this equation implies that the amplitude of rudder angle $\delta(t)$ is $|K_{eq}|$ times of that of error signal $e(t)$ and the phase angle of $\delta(t)$ is ahead of $e(t)$ by $\angle K_{eq}$. Besides, the fact that the describing function is independent of frequency ω but only dependent of the amplitude $\bar{\psi}$ of input signal must be kept in mind.

The function $G(j\omega)$ shown in fig. 8 is the frequency transfer function of ship with rudder angle regarded as the input and course angle as the output. Taking account of the fact that the input and the output of the hysteresis block representing the zigzag steering are $-\psi_0(t)$ and $\delta(t)$ respectively, it is able to be easily deduced from fig. 9 that the relation of eq. (4) is a sufficient condition for steady oscillatory state.

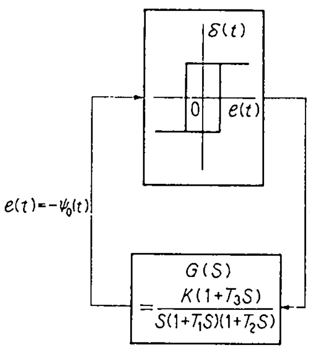


Fig. 9 Block diagram of limit cycle

$$G(j\omega)\cdot K_{eq}=-1 \quad (4)$$

Transforming this equation,

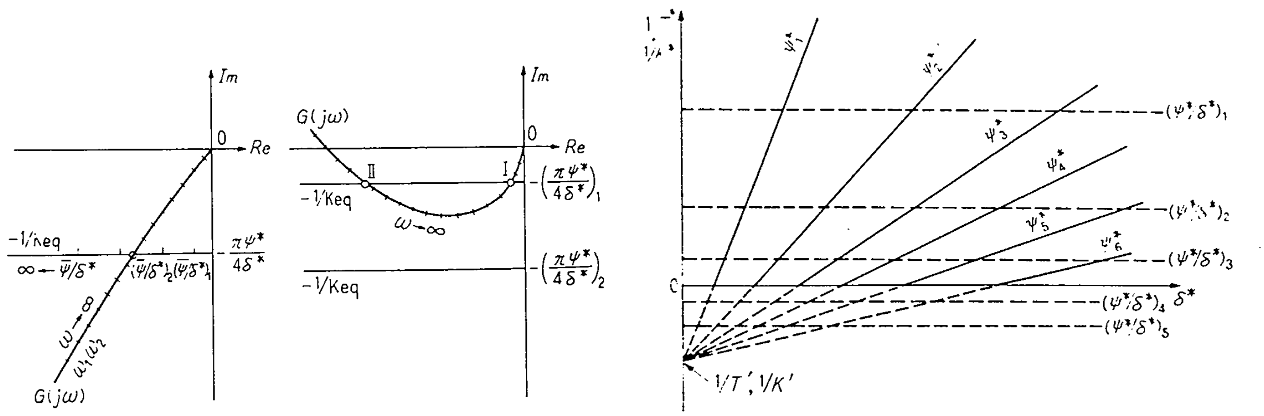
$$\left.\begin{aligned} R_e(G(j\omega))&=R_e\left(-\frac{1}{K_{eq}}\right) \\ I_m(G(j\omega))&=I_m\left(-\frac{1}{K_{eq}}\right) \end{aligned}\right\} \quad (5)$$

In other words, a cross point of the $G(j\omega)$ curve and $-1/K_{eq}$ curve on the Nyquist diagram corresponds to a steady oscillatory state, to say steady limit cycle.

By the way,

$$\left.\begin{aligned} R_e\left(-\frac{1}{K_{eq}}\right)&=-\frac{\pi\bar{\psi}}{4\delta^*}\cos\psi_1 \\ I_m\left(-\frac{1}{K_{eq}}\right)&=-\frac{\pi\psi^*}{4\delta^*} \end{aligned}\right\} \quad (6)$$

Accordingly, the $-1/K_{eq}$ curve is a half line parallel to the real axis of the Nyquist diagram. The distance from the half line to the real axis is uniquely determined by the combination of switching course angle ψ^* and rudder angle δ^*, to say $-\pi\psi^*/(4\delta^*)$. On the other hand, the behaviors of the frequency transfer function $G(j\omega)=K(1+T_3j\omega)/(j\omega\cdot(1+T_1j\omega)\cdot(1+T_2j\omega))$ on Nyquist diagram are different according to the course keeping qualities of ships. In fig. 10, are quan-



a) stable ship b) unstable ship c) T^* and K^* of an unstable ship

Fig. 10 Exsistance of limit cycle of zigzag maneuver

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titatively shown the behaviors of $G(j\omega)$ in two cases of stable and unstable ships.

(1) In case of stable ship. The $G(j\omega)$ curve always stays in the third quadrant of the coordinate plane without regard to variation of frequency ω. Namely, for infinitesimally small ω the absolute value of $G(j\omega)$ is infinitely large and vice versa. Therefore, the $G(j\omega)$ curve always intersects the half line representing $-1/K_{eq}$ at only one point as shown in fig. 10 a). Hence, the trajectory on the phase plane approaches gradually to the stable limit cycle corresponding to the cross point. As the frequency ω and the ratio $\bar{\psi}/\delta^*$ correspond to each point of the $G(j\omega)$ curve and the $-1/K_{eq}$ half line respectively, the amplitude $\bar{\psi}$ of course angle and the period τ of the limit cycle are automatically determined from the parameters, ω and $\bar{\psi}/\delta^*$, of these two curves at the cross point. Accordingly if the ratios of ψ^* to δ^*, to say ψ^*/δ^*, are equal to each other even for the different kinds of zigzag maneuver, for instance $20°\sim20°$, $10°\sim10°$ and $5°\sim5°$ zigzag maneuvers, their limit cycles have the same frequency ω. In this case, the amplitude $\bar{\psi}$ is strictly proportional to the switching course angle ψ^*.

(2) In case of unstable ship. The $G(j\omega)$ curve shifts from the third quadrant into the second quadrant as the frequency ω decreases less than a certain critical value, while the $-1/K_{eq}$ half line stays still in the third quadrant even in this case. Therefore, these two curves do not always intersect each other. There is no cross point in case where the switching course angle ψ^* is larger in comparison with the rudder angle δ^*. However, if the switching course angle ψ^* is diminished while the rudder angle δ^* is kept unchanged, the $G(j\omega)$ curve may intersect the $-1/K_{eq}$ half line as shown in fig. 10 (b). In this case, there usually exist two cross points. This implies that two steady oscillatory states may exist at the same time. After the inspection of stability of these two steady states, however, it is deduced that the steady state marked by II in the figure is unstable so that it can not be realized physically. From the reasoning stated above, it may be presumed that even if (10°, 10°) and (20°, 20°) normal zigzag maneuvers, where the former numbers in the parentheses mean the switching course angle ψ^* and the latter ones do the rudder angle δ^*, are unable to be executed in case of unstable ships, such modified zigzag maneuvers as (1°, 10°) or (2°, 20°) may be executed.

Similarly as in the figs. 5 and 6 in case of stable ships, in fig. 10 (c) are quantitatively shown the K^* and T^* values that will be determined from the first order system analysis of modified zigzag maneuvers in case of unstable ships. The value of the parameter ψ^*/δ^* corresponding to the abscissa of this figure is its critical value that decides whether it is possible to execute the modified zigzag maneuver.

1.6 A few examples of the modified zigzag maneuver.

Let's show the K^* and T^* values that were recently obtained by making use of two actual large ships. The principal dimensions of these ships are shown in Table 1.

Table 1 Principal dimensions of ships and the results of normal and modified zigzag maneuvers

		"A" ship	"B" ship
Length between perpendiculars, L_{pp} (m)		307.0	313.0
Moulded breadth, B (m)		48.2	48.2
Draft, d (m)		19.39	19.40
Displacement (ton)		250,750	250,251
Rudder area/$L_{pp}\times d$		1/69	1/66.7
(10°, 10°) normal zigzag maneuvers	$T_{68}'^*$	4.56	11.0
	$K_{68}'^*$	2.38	5.23
(1°, 5°) modified zigzag maneuvers	$T_{68}'^*$	1.13	3.62
	$K_{68}'^*$	0.927	2.17

There are full super tankers whose $r'-\delta'$ characteristics obtained by reverse spiral tests have the hysteresis loops representing

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unstable course keeping qualities as shown in figs. 11 and 12. In these figures, the time histories of (10°, 10°) normal zigzag maneuver

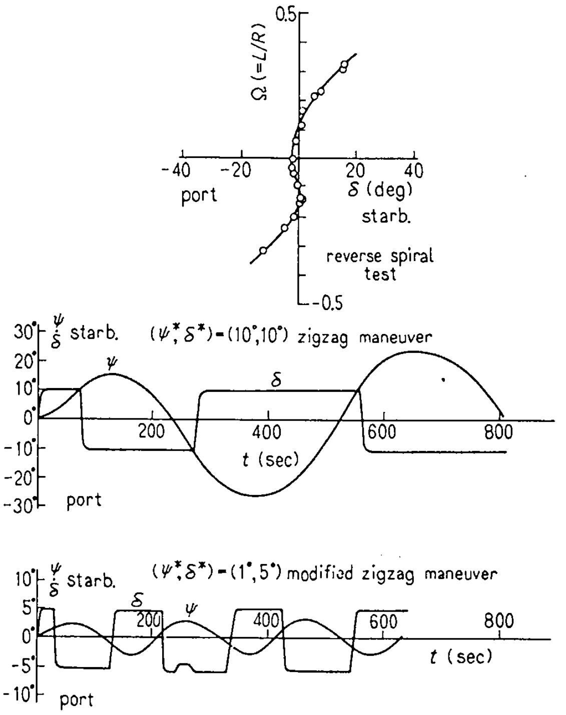

Fig. 11 Results of reverse spiral test and zigzag maneuver of "A" ship

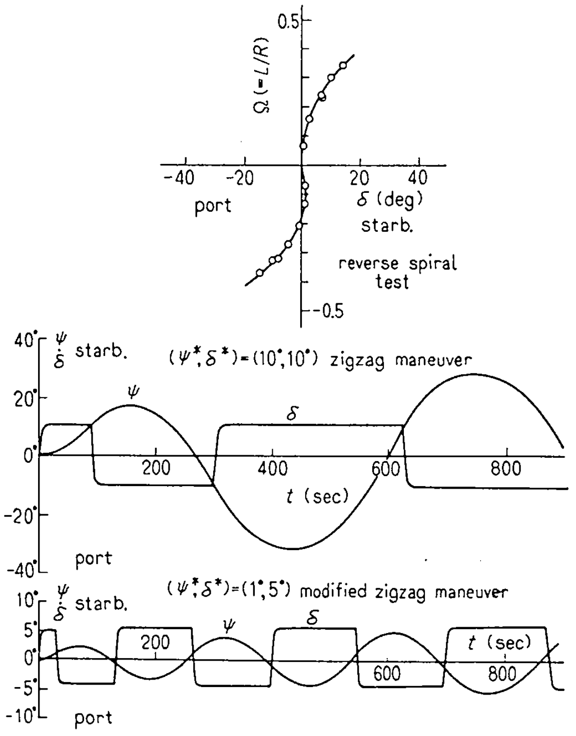

Fig. 12 Results of reverse spiral test and zigzag maneuver of "B" ship

and (1°, 5°) modified zigzag maneuver, to say $t-\psi$, δ curves, are also shown. The K^* and T^* values obtained from the first order system analysis of these time histories are tabulated in table 1 for comparison of each other. From the results, it may be deduced that the K^* and T^* values from (10°, 10°) normal zigzag maneuver are larger than those from (1°, 5°) modified one, so that the ships are of more stable course keeping quality at the modified zigzag maneuver than at the normal zigzag maneuver.

2. Application of Modified Zigzag Maneuver to Examination of the Frequency Respone Function

2.1 Determination of frequency response function by Fourier Analysis of limit cycles at the modified zigzag maneuver.

The direct method to obtain the frequency response function is Fourier analysis of the response to the sinusoidal steering of various frequencies ω. The modified zigzag maneuver is also able to be used to obtain the frequency response function, because the steady oscillatory states with various frequencies ω are able to be realized by interminable execution of modified zigzag maneuver[6]. The latter method is applicable even for unstable ships, while the former method, to say the sinusoidal steering is not applicable. This is the main reason that the modified zigzag maneuver have advantage over the sinusoidal steering.

However, the time histories of the rudder angle $\delta(t)$ and course angle $\psi(t)$ are not always sinusoidal ones at the zigzag maneuvers. Therefore, it is necessary to pick up the harmonic component with the fundamental frequency from these deformed signals of the rudder angle and course angle in order to obtain the frequedcy response function. The rudder angle of rectangular shape may be substituted by the equivalent sinusoidal function (7), of which the amplitude is equal to $4\delta^*/\pi$ and the period is the same as that of the rectangular rudder signal.

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$$\delta(t)=\frac{4\delta^*}{\pi}\sin\omega t \qquad (7)$$

As the fundamental harmonic component of the course angle $\psi(t)$ is $(4\delta^*/\pi)|G(j\omega)|\sin(\omega t+\angle G(j\omega))$, on the other hand, the absolute value $|G(j\omega)|$ and the phase angle $\angle G(j\omega)$ of the frequency response function $G(j\omega)$ are able to be determined from the amplitude and the phase angle of $\psi(t)$ at the steady oscillatory state. Hence, the frequency response function $H(j\omega)=K(1+T_3j\omega)/((1+T_1j\omega)(1+T_2j\omega))$ is calculated by using the following relationship.

$$|H(j\omega)|=\omega|G(j\omega)|, \quad \angle H(j\omega)=\angle G(j\omega)+\pi/2 \qquad (8)$$

In order to examine the validity of the above stated method, the frequency response function determined from the analysis of the various kinds of the modified zigzag maneuver which were executed on the analogue simulator will be compared with that from theoretical calculation by the relationship $H(j\omega)=K(1+T_3j\omega)/((1+T_1j\omega)(1+T_2\omega))$.

(1) Stable ship. The stability indices T_1', T_2', T_3' and K' used for the analogue simulation are shown in table 2, which were measured by means of forced yawing technique with a full super tanker[7].

Table 2 Hydrodynamic coefficients and stability indices of the ships used in the analysis of frequency response characteristics

	stable ship	unstable ship
$m'+m_y'$	26.2 ×10⁻³	26.2 ×10⁻³
Y_β'	20.0 〃	20.0 〃
$N_{\dot\beta}'$	0.09 〃	0.09 〃
N_β'	5.03 〃	5.03 〃
Y_r'	−10.78 〃	−10.78 〃
$Y_{\dot r}'$	− 0.437 〃	− 0.437 〃
N_r'	− 3.15 〃	− 2.30 〃
$I_z'+J_z'$	1.79 〃	1.79 〃
T_1'	12.68	−11.78
T_2'	0.420	0.484
T_3'	0.893	0.895
K'	5.82	− 6.20

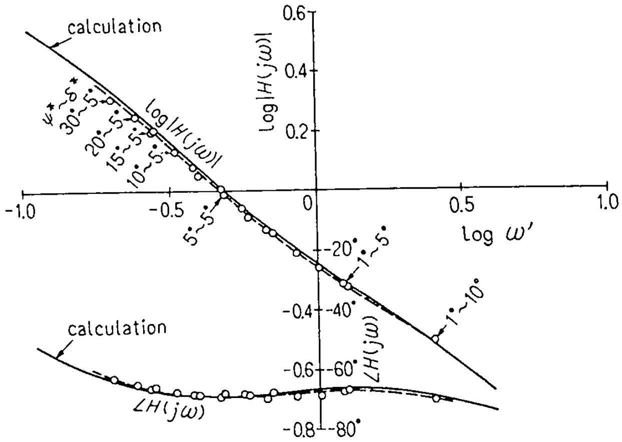

Fig. 13 Frequency response function of a stable ship obtained from various kind of modified zigzag maneuvers

Fig. 13 shows the frequency response functions thus obtained from the various modified zigzag maneuvers and the theoretical calculations. The abscissa of this figure is the common logarithm of dimensionless frequency $\omega'(=\omega L/U)$, to say $\log\omega'$, and the ordinate is common logarithm of the absolute value of the frequency response function $H(j\omega)$, to say $\log|H(j\omega)|$, or the phase angle $\angle H(j\omega)$. The dotted line connecting the circled points is the frequency response function obtained from the modified zigzag maneuvers, while the solid line is that from the calculations. These two lines agree with each other very well. This fact means that the modified zigzag maneuver is useful for obtaining the frequency response function. Inspecting these two lines in detail, however, it is found that the gain characteristics $|H(j\omega)|$ from the modified zigzag maneuver is slightly less than its true value. Even these differences between the measured values and the true values are negligibly small compared with the experimental errors of the zigzag maneuver at the sea.

(2) Unstable ship. As stated already in 1.5, the frequency response function can not be determined for the frequencies less than a certain critical frequency, at which the function $G(j\omega)$ posses its least imaginary value. However, the artificial deformation of the response function $G(j\omega)$ makes the

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modified zigzag maneuver possible to be executed, so that the frequency response function can be obtained even for the frequency less than the critical one. For the artificial deformation of the frequency response function, we shall make use of the circuit which makes the phase angle of its input signal advance. It is most desirable to choose the circuit whose gain characteristics decreases proportionally to the frequency ω, while the phase angle increases as the frequency ω decreases. As a matter of fact, however, the circuit as shown in fig. 15 will be used in the following study.

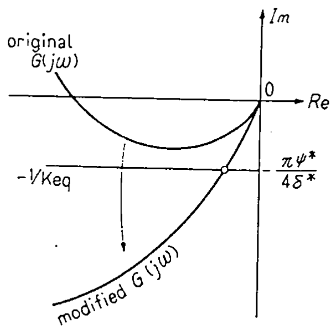

Fig. 14 Modification of frequency response function

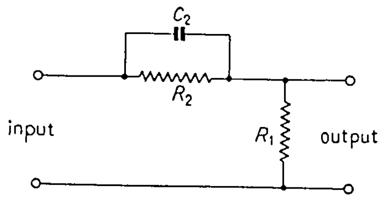

Fig. 15 Phase-shifting circuit

This circuit posses the maximum phase advance equal to $\tan^{-1}((1-\gamma)/(2\sqrt{\gamma}))$ at the frequency $\omega_0(=1/T_D\sqrt{\gamma})$. The response function of this circuit is calculated by eq. (9).[8]

$$\left.\begin{aligned} G_D(s)&=\gamma\cdot\frac{1+T_Ds}{1+\gamma T_Ds} \\ \gamma&=\frac{R_1}{R_1+R_2} \quad (0<\gamma<1) \\ T_D&=R_2\cdot C_2 \end{aligned}\right\} \tag{9}$$

By inserting this circuit after the block $G(s)$ as shown in fig. 16, that is to say the de-

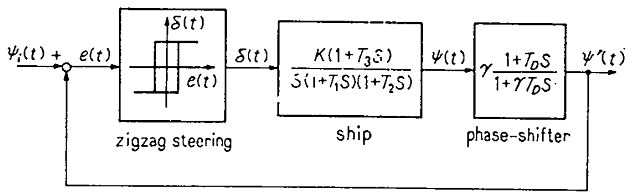

Fig. 16 Block diagram of zigzag maneuver in case of adopting phase-shifting circuit

formed course angle $\psi'(t)$ is fed back to the hysteresis block instead of the course angle $\psi(t)$, the frequency response function can be deformed as shown in fig. 14.

The resistances R_1, R_2 and the capacitance C_2 must be predetermined by adequately presuming the frequency ω_0 and the maximum phase advance. In fig. 17 is shown the analogue simulator with the phase shifting circuit, where the part enclosed by a dotted line is the phase shifting circuit. The frequency response function $G_D(j\omega)$ which is

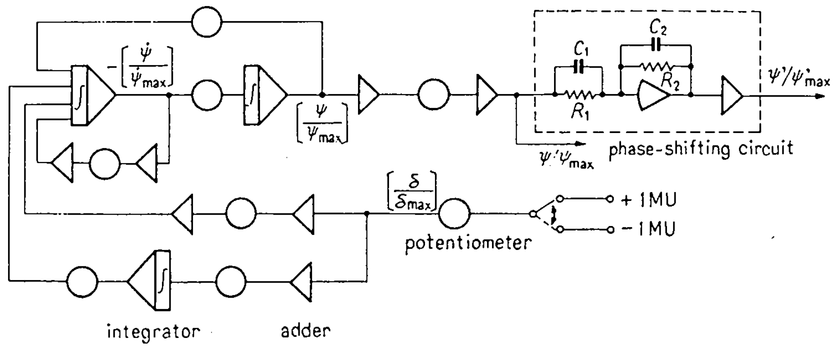

Fig. 17 Analogue simulator of zigzag meneuver in case of adopting phase-shifting circuit

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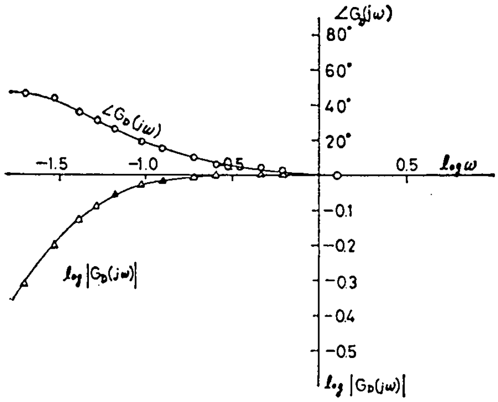

Fig. 18 Frequency response function of phase-shifting circuit

shown in fig. 18 was determined previously by the analysis of the responses to the various sinusoidal inputs.

The modified zigzag maneuver at which the artificial course angle $\psi'(t)$ is fed back instead of the real course angle $\psi(t)$, is turned to be realizable even for small frequency. Namely in case of the modified zigzag maneuver at which the real course angle $\psi(t)$ is fed back as hown in fig. 8, even the (1°, 5°) modified zigzag maneuver is unable to be executed, while the new modified zigzag maneuver at which the artificial course angle $\psi'(t)$ is fed back as shown in fig. 16, is able to be executed even for the remarkable small frequency. For instance, it is possible to execute even (5°, 5°), (10, 5°) and (20°, 5°) modified zigzag maneuvers where the former numbers in the parentheses mean the switching course angles in terms of the artificial course angle $\psi'(t)$ and the latter ones do the rudder angles.

From the frequency response function $G'(j\omega)$ determined by using the artificial course angle, the frequency response function $H(j\omega)$ which we want to know is calculated with ease by eq. (10)

$$\left.\begin{aligned}\log|H(j\omega)| &= \log\omega + \log|G'(j\omega)| - \log|G_D(j\omega)| \\ \angle H(j\omega) &= \angle G'(j\omega) - \angle G_D(j\omega) + \pi/2\end{aligned}\right\} \tag{10}$$

In order to examine the validity of the new modified zigzag maneuver, the stable ship quoted already in this paragraph will be turned to be unstable by changing only the yaw damping moment coefficient N_r' from -3.15×10^{-3} to -2.30×10^{-3}. The T_1', T_2', T_3' and K' indices of this artificial unstable ship are shown in table 2 as well as the hydrodynamic coefficients. The circled points and the points enclosed by triangles in fig. 19 represent the frequency respense functions

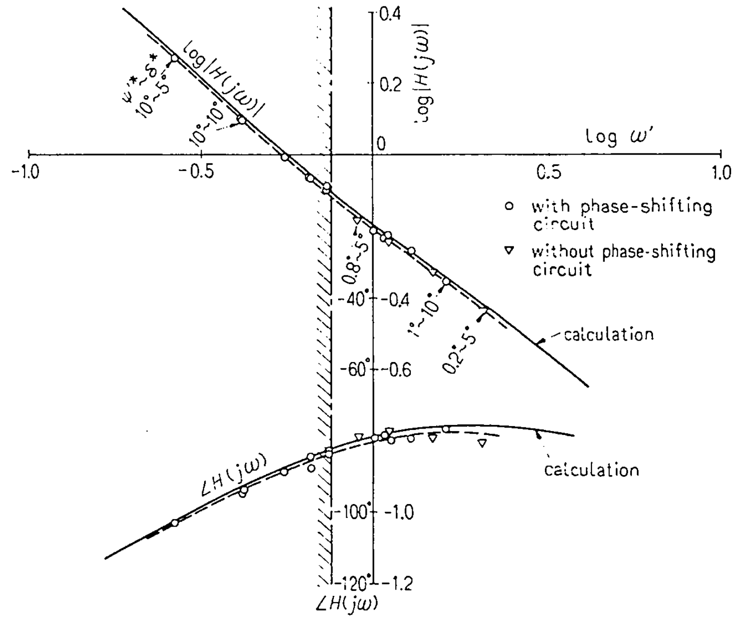

Fig. 19 Frequency response function of an unstable ship obtained from various kind of modified zigzag maneuvers

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obtained by the new modified zigzag maneuvers and the original modified ones respectively. These points agree well with the true response function shown by the solid lines in the figure.

At the region hatched by oblique lines where the logarithmic frequency, to say $\log\omega$, is less than -0.125, the original modified zigzag maneuvers is not realizable. As stated already, however, it is possible to execute the new modified zigzag maneuver even at the frequency within the region, so that the frequency response function is able to be obtained at the frequency whose logarithm is less than -0.125. The circled points in this fiure were obtained by using the artificial course angle $\psi'(t)$, to say the new modified zigzag maneuver and points enclosed by triangles were obtained by the original modified zigzag maneuvers. They agree well with each other as shown definitely in the Figure. However inspecting the points in detail, it will be found that the gain characteristics of the response function from the modified zigzag maneuvers are slightly smaller than the true values, and that the measured phase angles lag behined the true ones. Nevertheless, this method to obtain the frequency response function seems to be very useful because the differences between the measured values and the true ones are negligibly small.

2.2 Investigation of existance of limlt cycles in case of taking account of the nonlinearity of motion.

To estimate the steering motion of unstable ships correctly, it is necessary to take account of the nonlinearity of motion. Then in this paragraph, the conditions under which the existance of the limit cycle at the zigzag maneuver is assured, will be investigated.

Before begining the investigation of existance of limit cycle, it must be discussed in detail which nonlinear equation of motion is best representative of the steering motion of unstable ships. However in this study, the following nonlinear equation with the so-called cubic type nonlinearity which has been proposed by Nomoto[9] and Norrbin[10], will be used for the sake of brevity

$$T_1T_1\ddot{r}+(T_1+T_2)\dot{r}+r+pr^3=K\delta+KT_3\dot{\delta} \quad (11)$$

The rudder angle $\delta(t)$ at the steady oscillatory state, that is to say the limit cycle, can be described by the error signal $e(t)(\equiv-\psi(t))$ of course angle as follows;

$$\left.\begin{aligned} &\delta(t)=\alpha e(t)+\beta\dot{e}(t)=-\alpha\psi(t)-\beta\dot{\psi}(t)\\ &\alpha=\frac{4\delta^*}{\pi\bar{\psi}}\cos\psi_1,\quad \beta=-\frac{4\delta^*}{\pi\bar{\psi}\omega}\sin\psi_1,\\ &\psi_1=\sin^{-1}\left(\frac{\psi^*}{\bar{\psi}}\right)\end{aligned}\right\} \quad (12)$$

Substituting eq. (12) into eq. (11),

$$T_1T_2\dddot{\psi}+(T_1+T_2+KT_3\beta)\ddot{\psi} +(1+KT_3\alpha+K\beta)\dot{\psi}+p\dot{\psi}^3+K\alpha\psi=0 \quad (13)$$

The sinusoidal solution of this autonomous equation, to say $\bar{\psi}\sin\omega t$, gives the amplitude $\bar{\psi}$ and frequency ω of the limit cycle at the zigzag maneuver. Substituting $\bar{\psi}\sin\omega t$ into eq. (12) and equating the coefficients of the fundamental harmonic functions, $\sin\omega t$ and $\cos\omega t$, to zeros,

$$\left.\begin{aligned}&\left(\frac{3}{4}p\bar{\psi}^2-T_1T_2\right)\omega_2+1+KT_3\alpha+K\beta=0\\ &-(T_1+T_2+KT_3\beta)\omega^2+K\alpha=0\end{aligned}\right\} \quad (14)$$

The graphical solution that will be stated below seems to be useful for solving these equations, because this method is appropriate to reveal the close relation between the amplitude $\bar{\psi}$ and the frequency ω of the limit cycles, and the switching course angle ψ^* and the rudder angle δ^* of the zigzag steering. Rearranging eq. (14),

$$\frac{-\alpha}{\alpha^2+\beta^2\omega^2}=\frac{-K(1+T_3{}^2\omega^2)\left[T_1+T_2-T_3+\left(T_1T_2T_3-\frac{3}{4}pT_3\bar{\psi}^2\right)\omega^2\right]}{(1+T_1{}^2\omega^2)(1+T_2{}^2\omega^2)(1+T_3{}^2\omega^2)+F(\bar{\psi},\omega)}$$

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$$\frac{\beta\omega}{\alpha^2+\beta^2\omega^2}=\frac{-K(1+T_3^2\omega^2)\left[1+\left(T_1T_3+T_2T_3-T_1T_2+\frac{3}{4}p\bar{\psi}^2\right)\omega^2\right]}{\omega[(1+T_1^2\omega^2)(1+T_2^2\omega^2)(1+T_3^2\omega^2)+F(\bar{\psi},\omega)]} \tag{15}$$

$$F(\bar{\psi},\omega)=\left(\frac{9}{16}p^2T_3^2\bar{\psi}^4-\frac{3}{2}pT_1T_2T_3^2\bar{\psi}^2\right)\omega^6+\left(\frac{3}{2}p(T_3^2-T_1T_2)\bar{\psi}^2+\frac{9}{16}p^2\bar{\psi}^4\right)\omega^4+\frac{3}{2}p\bar{\psi}^2\omega^2$$

The left hand side term of the first equation of eq. (15) is $-\pi\bar{\psi}\cos\psi_1/(4\delta^*)$, and that of the second equation is equal to $-\pi\psi^*/(4\delta^*)$ respectively. Each of them is independent of the frequency ω. The right hand side term of the second equation is graphically represented by a curve as shown in fig. 20 if the parameter $\bar{\psi}$ is presumed previously. On the other hand, as the left hand side term of the second equation is constant independently of the frequency, it is represented by a half line parallel to the abscissa. The distance from the half line to the abscissa is uniquely determined by the equation $-\pi\psi^*/(4\delta^*)$. This half line always intersects the curves representing the right hand side term. A pair of $\bar{\psi}$ and ω corresponding to each of these cross points satisfy the second equation of eq. (15). One solid line as shown in fig. 21 is obtained for one prescribed value of ψ^*/δ^* by plotting a lot of the pairs, $(\bar{\psi},\omega)$ satisfying the second equation. Similarly, the left hand side term and the right hand side term of the first equation are represented by a half line parallel to the abscissa and certain curve respectively provided that the amplitude $\bar{\psi}$ is previously determined (see fig. 22). The cross point at which the half line intersects the curve representing the right hand side term determines a solution satisfying the first equation of eq. (15). The dotted line in fig. 21 is obtained by potting a lot of the pairs, $(\bar{\psi},\omega)$ corresponding to the cross points of fig. 22. It is obvious from the reasoning stated above that the cross point of the solid line and the dotted line in fig. 21 is representative of a limit cycle at the zigzag maneuver with the prescribed value of ψ^*/δ^*. The ordinate and abscissa of the cross point are the am-

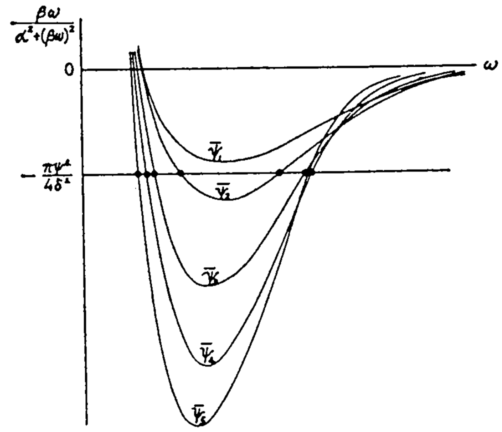

Fig. 20

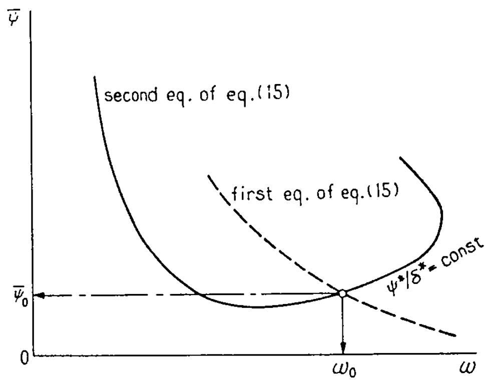

Fig. 21

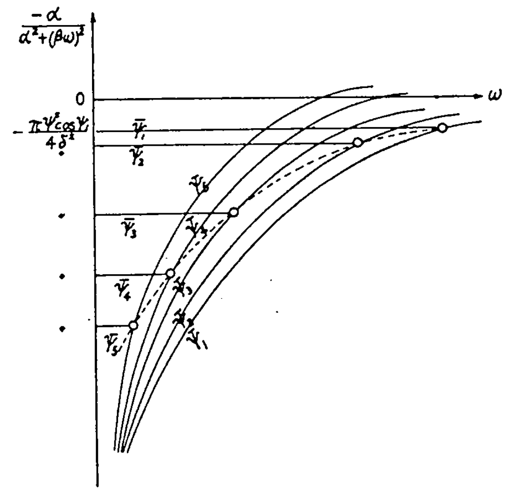

Fig. 22

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plitude $\bar{\psi}$ of course angle and the frequency ω of the limit cycle respectively.

As an example of the graphical solution, the modified zigzag maneuvers of the unstable ship will be studied by assuming the so-called cubic type nonlinearity, the coefficient p' of which is equal to -20.0. Fig. 23 shows the $r'\sim\delta'$ characteristics of the unstable ship. Some pairs of the amplitude $\bar{\psi}$ and the frequency ω, to say $(\bar{\psi}, \omega)$, of the limit cycles are shown in fig. 24 for the various kinds of ψ^* and δ^*, where the circled points are the graphical solutions. The dotted and solid lines are, as stated above, representative of the first and the second equations of eq. (15) respectively. The points enclosed by triangles, on the other hand, mean the limit cycles obtained by the analogue simulation of the modified zigzag maneuver.

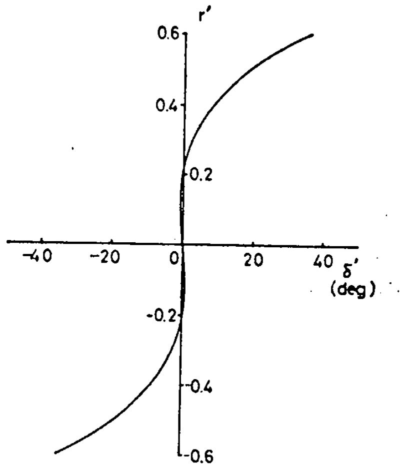

Fig. 23 $r'\sim\delta'$ characteristics of an unstable ship used at the nonlinear analysis of modified zigzag maneuver ($T_1'=-11.78$, $T_2'=0.484$, $T_3'=0.895$, $K'=-6.20$, $p'=-20.0$)

The graphical solutions do not well agree with the results from the analogue simulation. However, it is evident from this figure that both of the circled points and the triangular points locate on the same dotted line representing the first equation. Consequently, if the solid line which is representative of the second equation were brought down up to the broken line, the graphical solution should agree with the results of the analogue simulation. In spite of the inaccuracy, the graphical solutions seem to useful for studying the limit cycle quantitatively. By the way, comparing the results obtained by taking account of the nonlinearity of steering motion with those from the linear analysis, the following facts may be deduced;

(1) In the case where the equation of motion is linear, the frequency of the limit cycle is uniquely determined by the ratio ψ^*/δ^*, independently of the values of ψ^* and

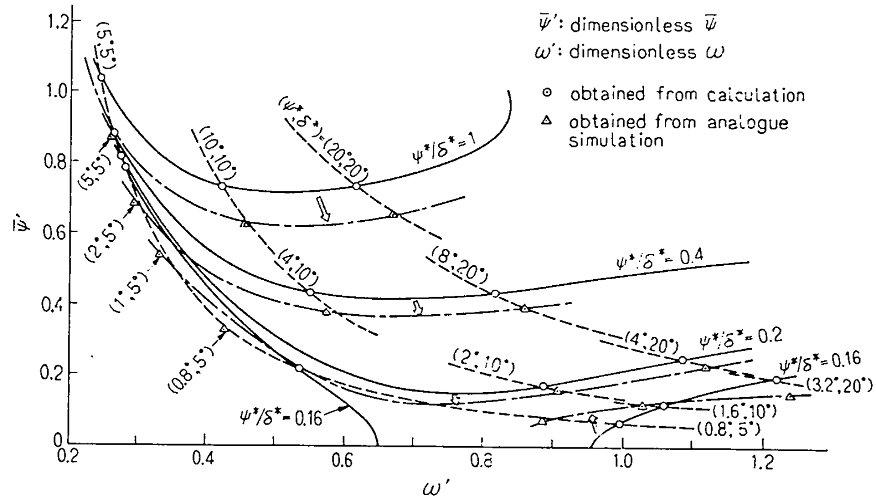

Fig. 24 Amplitude of head angle $\bar{\psi}$ and frequency ω' of limit cycles at the various kind of modified zigzag maneuvers carried out with an unstable ship in consideration of nonlinearity

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δ^*, and moreover the amplitude $\bar{\psi}$ is strictly proportional to the switching angle ψ^*. However, this statement is not true in case of the nonlinear equation of motion. The frequency of the limit cycle increases as the values of ψ^* and δ^* increase, though the ratio ψ^*/δ^* is kept unchanged. The amplitude $\bar{\psi}$ possesses its minimum value at certain pair of ψ^* and δ^* as shown in fig. 24.

(2) In case of nonlinear equation of motion, there exist more than one stable limit cycles at certain kinds of ψ^* and δ^*, to say ψ^*/δ^* equal to 0.16. The (0.8°, 5°) modified zigzag maneuver, for instance, has three limit cycles while the (1.6°, 10°) modified zigzag maneuver has only one limit cycle. However, all of these three limit cycles are not stable. The limit cycle whose ($\bar{\psi}'$, ω') value is (0.23, 0.52) is unstable and unrealizable, while the remaining two limit cycles whose ($\bar{\psi}'$, ω') values are (0.073, 1.002) and (0.78, 0.28) respectively, are stable and realizable. The former limit cycle of these two stable ones is usually realized if the zigzag maneuver is started from the straight course.

Conclusion

Summing up the discussions stated above, the following conclusions will be deduced;

(1) In case of less stable or unstable ships, it is difficult or impossible to execute the normal zigzag maneuver at small rudder angle. However at the modified zigzag maneuver where the switching course angle is small than the rudder angle, even such less stable or unstable ships can realize the stable limit cycles. Therefore, the modified zigzag maneuver is very useful for examining the course keeping quality, whether a ship is stable or not.

(2) It is not appropriate to describe all kinds of the steering modes by only one first order approximation. As proposed by Nomoto and Karasuno[11], it is necessary to describe the steering motion with the second order differential equation taking account of the nonlinearity. However if the principal modes of the steering motion are classified into a) course keeping maneuver, b) course change maneuver and c) emergency maneuver, and if they are separately treated, the first order system approximation is still useful because of simplicity. In this case, the different K and T indices must be used properly according to the relevant steering mode. Namely, for b) and c) modes, the K and T indices obtained from (15°, 15°) and (35°, 35°) normal zigzag maneuvers respectively are appropriate, while for a) mode those from (1°, 5°) modified zigzag maneuver are suitable.

(3) Even in case of unstable ships, the stable limit cycles are realized if the ratio of the switching course angle ψ^* to the rudder angle δ^* is less than a certain value. Nevertheless, there exists no stable limit cycle with the frequency less than a certain critical value. If the artificial course angle whose phase angle leads in advance of the real course angle is fed back instead of the real course angle, it is possible to execute the modified zigzag maneuver with the large switching course angle, so that the stable limit cycle is able to be realized even at the frequency less than the above stated critical value.

(4) By analysing the response to the zigzag steering after the limit cycle is reached, the frequency response function of maneuvering motion can be obtained with enough accuracy. This method is by far better than the sinusoidal steering, because it is applicable even for the unstable ships.

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