A study on the wave-induced ship-hull vibration: springing caused by higher-order wave exciting force

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ABSTRACT

This paper deals with springing caused by higher-order exciting forces both theoretically and experimentally.

Many studies have been carried out in this field; however, there seems to be some room for further improvements in evaluation method of exciting forces and experimental technique.

This paper proposes an analytical method for calculating higher-order exciting forces and an experimental technique for investigating dynamic behaviours of an elastic ship in waves.

1. INTRODUCTION

A ship is a moving elastic body; therefore, it is subjected to ship motion (response as a rigid body) and ship hull vibration (response as an elastic body) by wave load simultaneously.

With the increase in size of ships and the resultant decrease in their rigidity, wave-induced ship-hull vibration has become remarkably serious even in comparatively calm sea conditions. This phenomena is called "Springing" and has been studied by many researchers.[1] ∿ [10]

The cause of springing has been considered mainly in two ways. One is the resonance with linear exciting force, and the other is the superharmonic resonance with higher-order exciting force. Many experimental data which have been reported till now cannot be fully explained only by linear exciting force and suggest the role of higher-order exciting force on springing. [3] [4] [6] [10]

Some calculation methods which are based on "Momentum theory" have been proposed concerning the exciting force related to springing, but these methods are not satisfactory from hydrodynamic point of view, especially in free surface condition as Tasai has pointed out. [1] Moreover, in the experimental technique, there seems to be some room for further improvement.

What we have carried out in this study are;

1) To confirm springing phenomena, especially higher-order resonance in a series of experiment using an elastic model.
2) To calculate the second order exciting force by solving two dimensional wave making problem.
3) To apply this hydrodynamic force to the analysis of ship-hull vibration, and
4) To compare the calculated vibratory response with the experimental data.

The main symbols used in this paper are as follows.

f_n : natural frequency of n'th mode
(f_1 represents the natural frequency of two-noded vibration)
ω_n : $2\pi f_n$
f_e : encounter wave frequency
ω_e : $2\pi f_e$
f_w : wave frequency
ω : $2\pi f_w$
V : ship speed
$w(x)$: weight per unit length
$m_a(x)$: added mass per unit length
ρ : density of water
$K = \omega^2/g$
$K_e = \omega_e^2/g$
$a1$: vertical acceleration in fore part
$a2$: vertical acceleration in midship section
M : longitudinal bending moment in midship section
ζ_A : amplitude of encounter wave
g : acceleration of gravity

2. MODEL EXPERIMENT

Until now, many experimental studies have been attempted. The models used in these studies consist of an elastic beam and several ship's sections connected to the beam. [3] ∿ [7]

However, it is not always easy to measure the elastic behaviours with this type of models. The difficulties lie in the type of pick-up, the method of connecting the sections, the water-tightness of joints of sections, etc.

In order to overcome these defects, a solid body made of hard rubber was used in this study for the towing tests in head waves.

Through the experiments, the relations between encounter wave and vibratory responses such as acceleration, bottom pressure, wave bending moment in midship section, etc. have been measured.

2.1 Model

Principle particulars of the model are shown in Table 1, and the arrangement of the model and measuring instruments are indicated in Fig.1. The model was towed at two points by guides.

We paid careful attention to similarity law and the choice of material.

Similarity of model It is impossible to make a model completely similar to an actual ship in all aspects; therefore, the model has been designed taking mainly "similarity in time" into account.

Materials From the view point of properties of material, rubber is different from steel. However, the authors consider it possible to employ the model made of hard rubber if its linearity and inner damping are known. Therefore, bending test in the air and free vibration test in the towing condition were carried out. The results of the former test were used for the calibration of longitudinal bending moment, and the damping coefficient given from the latter test was used for the analysis of vibration.

2.2 Contents of Experiment

All measurements were made in regular head waves. The items measured at the experiment are vertical acceleration in fore part and midship section, pressure on ship's bottom, longitudinal bending moment, and wave height. At the same time, ship motion (pitching, rolling) and wave profile on the hull surface were measured.

The damping coefficient and the natural frequency were found by giving the model a shock to produce free damped vibration in towing condition.

The results are shown in Table 1.

2.3 Results of Experiment

Some examples of recorded signals are shown in Fig.2 and Fig.3. These data were obtained in the case $f_e = f_1/m$ (m=2,3, ...) and superharmonic resonance is found.

Any signal $x(t)$ is represented by Fourier series with finite terms, $\tilde{x}(t)$, as follows.

$$\tilde{x}(t) = \frac{X_0}{2} + \sum_{k=1}^{N/2-1} X_k \cos(2\pi f_k t + \alpha_k) + \frac{X_{N/2}}{2} \cos 2\pi f_{N/2} t$$

Examples of X_k and α_k are given in Figs.4 and 5 for $f_e = f_1/2$ and $f_e = f_1/3$ respectively.

Encounter wave includes the basic frequency component and the double frequency component. This means that in considering the second order problem, wave surface cannot be treated as the complete sine wave. This can be explained theoretically (as mentioned later).

As for longitudinal bending moment, M, bottom pressure, P2, P3, P5, vertical acceleration at fore part, a_1, and vertical

Table 1. Principal particulars of the model

Length	L	3.80 m
Breadth	B= 2b	0.50 m
Depth	D	0.30 m
Draft	d	0.10 m
Displacement	W	172.00 kg
Midship Coefficient	$C_{\boxtimes}$	1.00
Longitudinal Gyradius	κ	1.96 m
Center of Gravity from Midship	X_G	0.00 m
[Measured Value]		
Natural Frequency of Two-Noded Vibration	f_1	3.5 Hz
Logarithmic Decrement	Λ_1	0.241 (Fn=0.0) 0.203 (Fn=0.066) 0.197 (Fn=0.098) 0.197 (Fn=0.131) 0.193 (Fn=0.164)

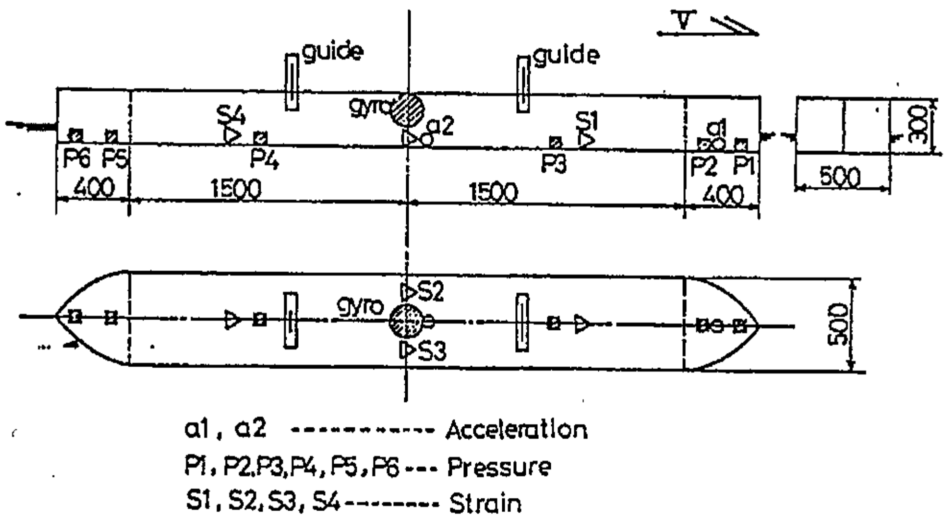


Fig.1 Arrangement of model and measuring instruments

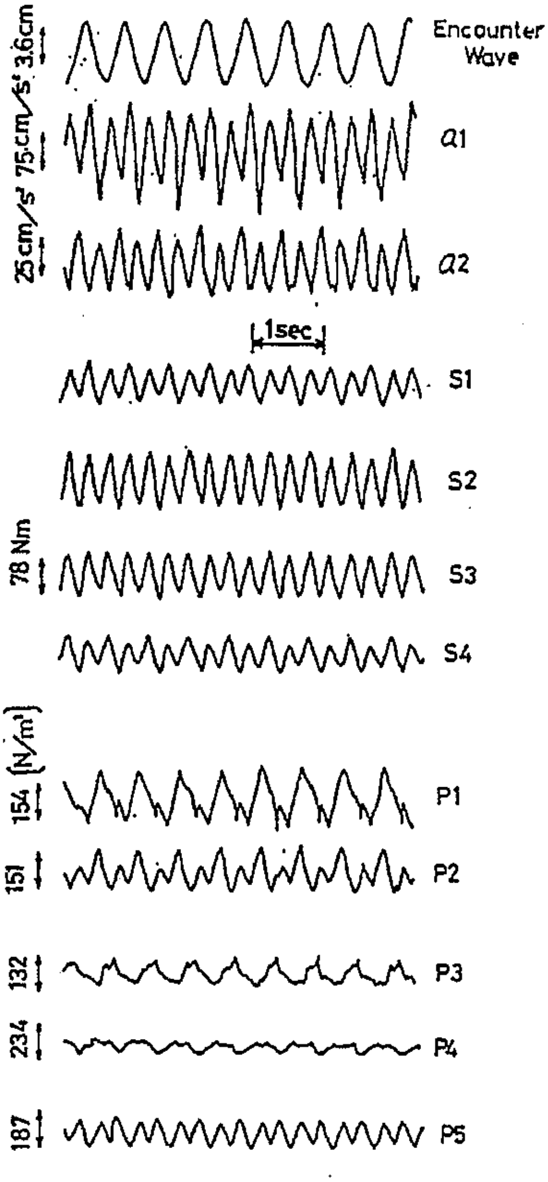


Fig.2 Examples of recorded signals in case of $f_e = f_1/2$ (f_w =1.12Hz , f_e =1.75Hz , V = 0.8 m/s , λ =1.24m)

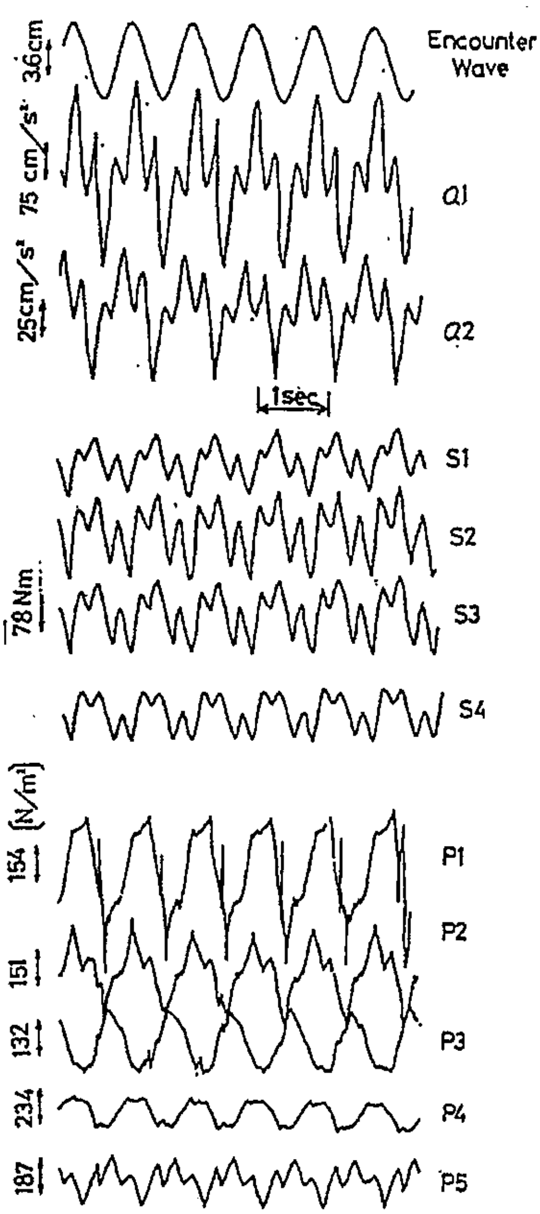


Fig.3 Examples of recorded signals in case of $f_e = f_1/3$ (f_w=0.83Hz , f_e=1.16Hz , V=0.8m/s , λ= 2.26m)

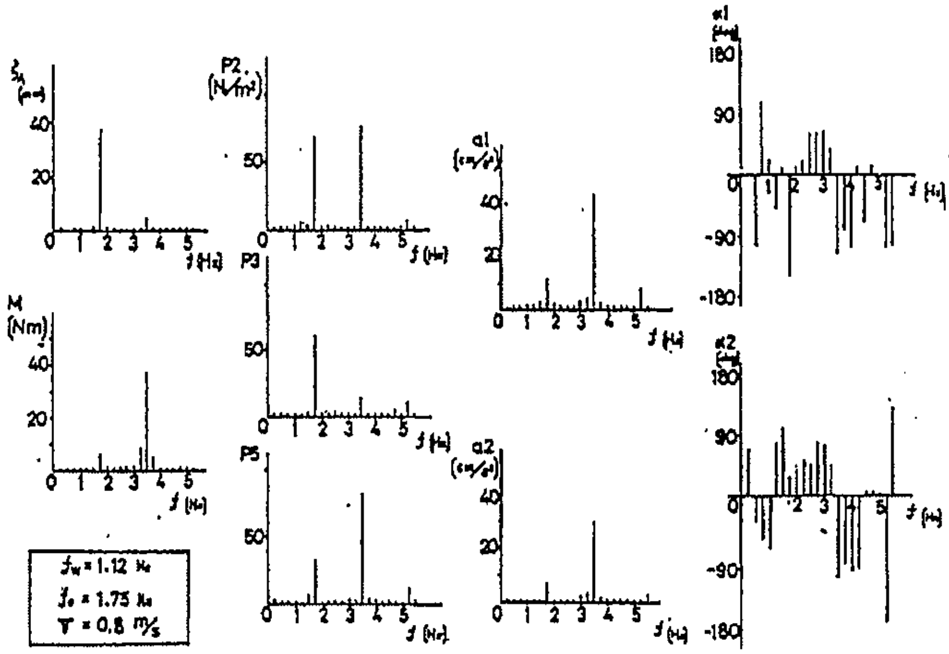


Fig. 4 An example of Fourier spectrum ($f_e = f_1/2$)

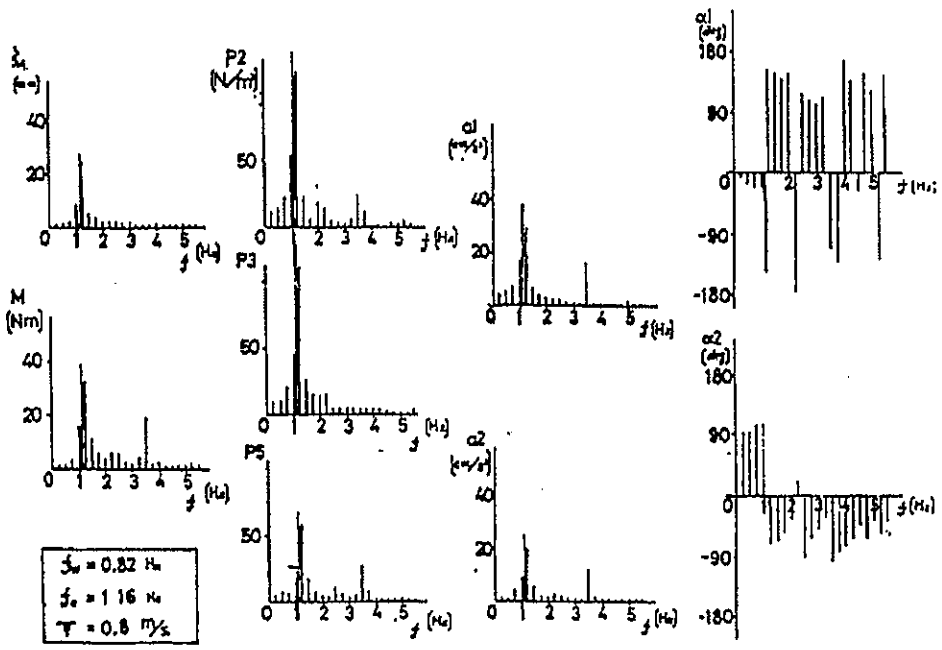


Fig. 5 An example of Fourier spectrum ($f_e = f_1/3$)

acceleration at midship section, $a2$; two peaks are found in the components corresponding to the encounter frequency and to the natural frequency of two-noded vibration.

The bottom pressure, P2 , is greater than P5 in the encounter wave frequency, but there is little difference between P2 and P5 at f_1-frequency.

By changing encounter wave frequency systematically, the variation in the vibratory responses can be found. Fig.6 shows an example of the variation of a component oscillating at frequency f_1 .

3. EVALUATION OF HIGHER-ORDER EXCITING FORCE

The model experiments have confirmed that superharmonic resonance occurs even if pitching amplitude is negligibly small. This suggests that springing phenomena is not merely the first order resonance with encounter wave.

In this paper, the second order exciting force has been evaluated by perturbation method. Strictly speaking, wave exciting force in head waves should be solved as a three dimensional problem. In this paper, however, calculation has been carried out in the same way as strip theory supposing that the phenomena is two dimensional. From a physical point of view, this calculation method assumes that reflected wave radiates at right angles to longitudinal direction, and this assumption would be acceptable in treating this sort of phenomena.

3.1 Coordinate System

Coordinate systems used in this paper are body-fixed one, o-xyz , and space fixed one, o'- x'y'z' , as shown in Fig.7.

Let the velocity potential of incident wave be

$$\Phi_I(x', y', z', t) = \frac{g\zeta_A}{\omega} e^{kz'} \cos(\omega t + kx') \qquad (1)$$

then, the free surface elevation is given as follows.

$$\zeta = -\frac{1}{g}\left(\frac{\partial \Phi_I}{\partial t} + \frac{1}{2}\nabla\Phi_I \cdot \nabla\Phi_I\right)_{z=\zeta}$$
$$= \zeta_A \sin(\omega_e t + kx) - \frac{1}{2} k\zeta_A^2 \cos(2\omega_e t + 2kx) + \cdots \qquad (2)$$

The ratio of the second order elevation to the first order one is shown in Fig.8.

Similarly, sub-surface is given as follows.

$$\zeta_S = \zeta_{SA} \sin(\omega_e t + kx) - \frac{1}{2} k\zeta_{SA}^2 \cos(2\omega_e t + 2kx) + \cdots \qquad (3)$$

where $\zeta_{SA} = \zeta_A e^{-kd}$

3.2 Formulation of Problem

Assuming the problem as two dimensional, diffraction force is estimated from the solution of radiation problem; that is, the vertical component of the orbital velocity of incident wave at the ship's bottom which is fixed in wave, is considered as the heaving velocity of a two-dimensional body

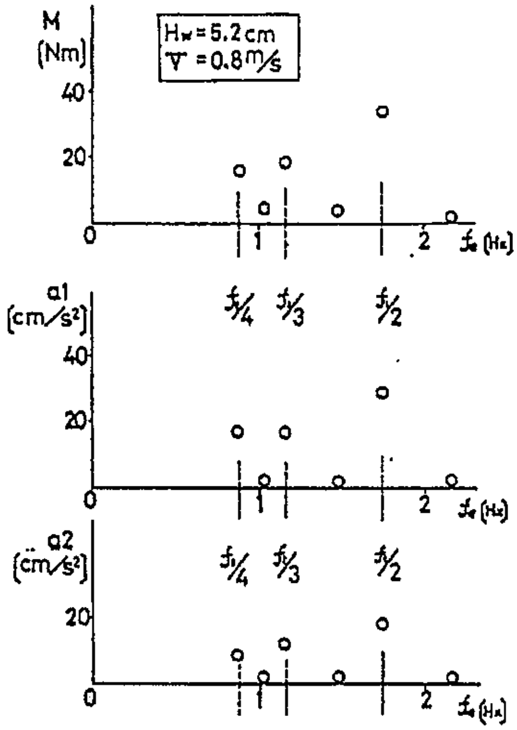


Fig.6 An example of superharmonic resonance : variation of a component oscillating at frequency f_1

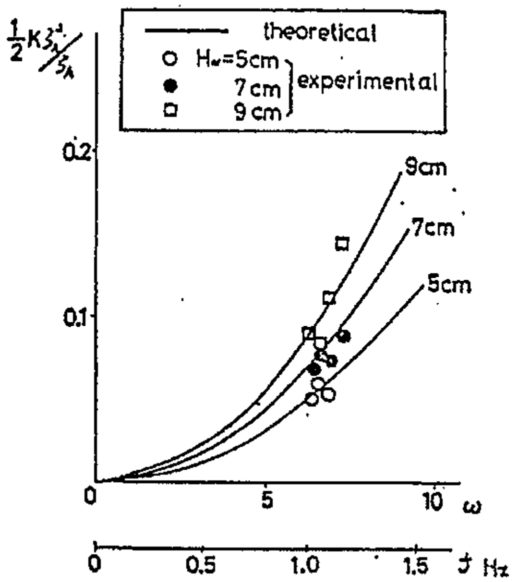


Fig.8 Comparison between the first-order surface elevation and the second-order one

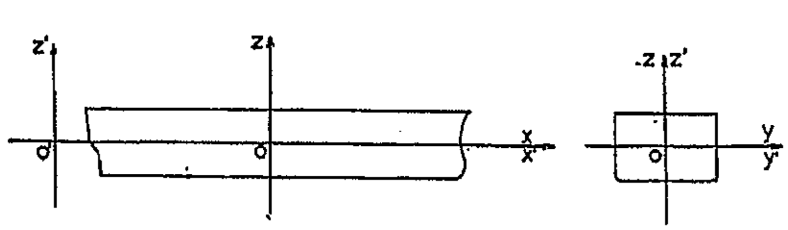


Fig.7 Coordinate axes

oscillating in a free surface.

Thus, substituting radiation problem for diffraction problem, diffraction force is evaluated in what follows.

The problem is formulated as follows.

$$\nabla^2\Phi(y,z,t) = \left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi(y,z,t) = 0 \quad \text{in fluid} \tag{4}$$

$$\Phi_{tt}(y,z,t)+g\Phi_z+2\Phi_y\Phi_{ty}+2\Phi_z\Phi_{tz}+\Phi_y^2\Phi_{yy}+2\Phi_y\Phi_z\Phi_{yz}+\Phi_z^2\Phi_{zz}=0 \quad \text{on } z=\zeta(y,t) \tag{5}$$

$$\frac{\partial\Phi}{\partial n} = -\left(\frac{\partial}{\partial t}-V\frac{\partial}{\partial z}\right)\zeta_s\cdot\frac{\partial z}{\partial n} \quad \text{on } z=-d \tag{6}$$

$$\Phi(y,z,t) = \Phi(-y,z,t) \tag{7}$$

$$\Phi_z(y,z,t) = 0 \quad \text{as } z\to-\infty \tag{8}$$

$$\frac{\partial\Phi}{\partial y} \pm ik_e\Phi = 0 \quad \text{as } y\to\pm\infty \tag{9}$$

Several authors such as Potash [11], Lee [12][13], Kim [14], and Yamashita [15] have reported the fundamental studies of second order radiation problem.

The calculation method employed in this paper is in accordance with Lee's paper, and symbols are mostly the same as those used in it.

Assuming wave amplitude to be small, the velocity potential is expanded as follows;

$$\Phi(y,z,t) = Re\left\{\sum_{n=1}^{\infty}\sum_{m=0}^{n}\varepsilon^n\varphi_m^{(n)}(y,z)e^{-im\omega_e t}\right\} \tag{10}$$

$$\varphi_m^{(n)}(y,z) = \varphi_{mc}^{(n)}(y,z)+i\varphi_{ms}^{(n)}(y,z)$$

Similarly, the free surface elevation is expanded as follows;

$$\zeta = \sum_{n=1}^{\infty}\varepsilon^n\{\zeta_0^{(n)}(y)+\zeta^{(n)}(y,t)\} \tag{11}$$

while the body surface is given by

$$S_0(y_0,z_0) = f(y_0)-z_0 = 0$$

Substituting (10) and (11) into the equations (4) ∿ (9), and collecting the terms of the same order, five sets of equations are derived up to the second order. However, by brief consideration, the equations to be solved can be reduced to the following one.

<u>1st order</u>

Let $\varphi^{(1)}(y,z) = \varphi_1^{(1)}(y,z) = \varphi_{1c}^{(1)}(y,z)+i\varphi_{1s}^{(1)}(y,z)$

then,

$$\nabla^2\varphi^{(1)}(y,z) = 0 \quad \text{outside } S_0 \tag{12}$$

$$\varphi_z^{(1)}(y,0) - K_e\varphi^{(1)}(y,0) = 0 \tag{13}$$

$$\varphi_y^{(1)}f' - \varphi_z^{(1)} = -b\omega \quad \text{on } S_0 \tag{14}$$

$$\varphi_z^{(1)}(y,-\infty) = 0 \tag{15}$$

$$\lim_{y\to\pm\infty}(\varphi_y^{(1)} \mp iK_e\varphi^{(1)}) = 0 \tag{16}$$

2nd order

Let $\varphi^{(2)}(y,z) = \varphi_2^{(2)}(y,z) = \varphi_{2c}^{(2)}(y,z) + i\,\varphi_{2s}^{(2)}(y,z)$

then, $\nabla^2 \varphi^{(2)}(y,z) = 0$ outside S_0 (17)

$$\varphi_z^{(2)}(y,0) - 4K_e\varphi^{(2)}(y,0)$$
$$= \frac{\omega_e}{2g}\{\varphi_{1s}^{(1)}(y,0)(\varphi_{1czz}^{(1)} - K_e\varphi_{1cz}^{(1)}) + \varphi_{1c}^{(1)}(\varphi_{1szz}^{(1)} - K_e\varphi_{1sz}^{(1)})$$
$$- 4(\varphi_{1cy}^{(1)}\varphi_{1sy}^{(1)} + \varphi_{1cz}^{(1)}\varphi_{1sz}^{(1)})\}$$
$$+ i\frac{\omega_e}{2g}\{\varphi_{1s}^{(1)}(y,0)(\varphi_{1szz}^{(1)} - K_e\varphi_{1sz}^{(1)}) - \varphi_{1c}^{(1)}(\varphi_{1czz}^{(1)} - K_e\varphi_{1cz}^{(1)})$$
$$+ 2(\varphi_{1cy}^{(1)2} - \varphi_{1sy}^{(1)2}) + 2(\varphi_{1cz}^{(1)2} - \varphi_{1sz}^{(1)2})\} \quad (18)$$

$$\varphi_y^{(2)}f' - \varphi_z^{(2)} = \frac{b}{2}(\varphi_{1szy}^{(1)}f' - \varphi_{1szz}^{(1)})$$
$$- i\frac{b}{2}(\varphi_{1czy}^{(1)}f' - \varphi_{1czz}^{(1)} + b^2\omega K) \quad \text{on } S_0 \quad (19)$$

$$\varphi_z^{(2)}(y,-\infty) = 0 \quad (20)$$

$$\lim_{y\to\pm\infty}(\varphi_y^{(2)} \pm i4K_e\varphi^{(2)}) = 0 \quad (21)$$

3.3 Velocity Potential and Pressure

Assuming that ship section is mapped from a circular cylinder with unit radius, it is sufficient to solve the problem in respect to this reference circle. The approach to the solution is as follows. The first step is to place at the origin a source and multiples that satisfy the boundary conditions at free surface, sea bottom, and infinity. And the second step is to determine the strength of the source and multiples so as to satisfy the boundary condition on the body. For mapping, Lewis Form is used.

The solution in the first order is well known as the foundation of Strip Theory.

Though the free surface condition in the second order is non-homogeneous, it is solved in the same way as the first order, because the particular solution has been already known [16].

Letting $p(y,z,t) = p_0(y,z) + \varepsilon p^{(1)}(y,z,t) + \varepsilon^2 p^{(2)}(y,z,t)$

then, $p^{(1)}(y_0,z_0,t) = -\rho\{\omega_e\varphi_{1s}^{(1)}(y_0,z_0)\cos\omega_e t + (gb - \omega_e\varphi_{1c}^{(1)})\sin\omega_e t\}$ (22)

$$p^{(2)}(y_0,z_0,t) = \frac{\rho b\omega_e}{2}\varphi_{1cz}^{(1)}(y_0,z_0) - \frac{\rho}{4}(\varphi_{1cy}^{(1)2} + \varphi_{1sy}^{(1)2} + \varphi_{1cz}^{(1)2} + \varphi_{1sz}^{(1)2})$$
$$- \rho\{2\omega_e\varphi_{2s}^{(2)}(y_0,z_0) + \frac{b\omega_e}{2}\varphi_{1cz}^{(1)}$$
$$+ \frac{1}{4}(\varphi_{1cy}^{(1)2} - \varphi_{1sy}^{(1)2} + \varphi_{1cz}^{(1)2} - \varphi_{1sz}^{(1)2})\}\cos 2\omega_e t$$
$$+ \rho\{2\omega_e\varphi_{2c}^{(2)}(y_0,z_0) - \frac{b\omega_e}{2}\varphi_{1sz}^{(1)}$$
$$- \frac{1}{2}(\varphi_{1cy}^{(1)}\varphi_{1sy}^{(1)} + \varphi_{1cz}^{(1)}\varphi_{1sz}^{(1)})\}\sin 2\omega_e t \quad (23)$$

4. SHIP-HULL VIBRATION INDUCED BY THE SECOND-ORDER EXCITING FORCE

4.1 Equation of Vibration and Its Solution

The amplitude of vibratory responses (vertical acceleration, longitudinal bending moment, pressure) is evaluated in the case where the encounter frequency is a half of the natural frequency of vertical two-noded vibration, that is, $f_e = f_1/2$.

Let the dynamic deflection be represented by the summation of the modal components as follows.

$$Z(x,t) = \sum_{n=1}^{\infty} A_n(t) X_n(x) \tag{24}$$

where $A_n(t)$ is the modal amplitude and $X_n(x)$ is the characteristic shape. Then Lagrange's equation is given by

$$\frac{d}{dt}\left(\frac{\partial K_m}{\partial \dot{A}_n}\right) + \frac{\partial \mathcal{U}}{\partial A_n} - \frac{\partial W_c}{\partial A_n} = \frac{\partial W_e}{\partial A_n} \tag{25}$$

where

$$K_m = \frac{1}{2}\int_{-l/2}^{l/2}\left\{\frac{w(x)}{g} + m_a(x)\right\}\left[\sum_{n=1}^{\infty}\dot{A}_n^2(t) X_n^2(x)\right] dx \tag{26}$$

: kinematic energy of the complete system

$\mathcal{U}$: strain energy

W_c : work done by damping force

$$W_e = \int_{-l/2}^{l/2} F_W(x,t)\left[\sum_{n=1}^{\infty} A_n(t) X_n(x)\right] dx \tag{27}$$

: work done by external dynamic force

$$F_W(x,t) = F_{W2}(x)\sin(2\omega_e t + \varepsilon_2(x)) \tag{28}$$

: the second order wave exciting force which induces ship-hull vibration

The equivalent force is represented as follows.

$$\int_{-l/2}^{l/2} F_W(x,t) X_n(x) dx = F\sin(2\omega_e t + \delta_1) \tag{29}$$

Then, the modal amplitude is derived as follows.

$$A_n(t) = \frac{F\sin(2\omega_e t + \delta)}{\omega_n^2\left[\int\left\{\frac{w(x)}{g} + m_a(x)\right\} X_n^2(x) dx\right]\sqrt{\left\{1 - \frac{(2\omega_e)^2}{\omega_n^2}\right\}^2 + \left\{2C_n\frac{(2\omega_e)}{\omega_n^2}\right\}^2}} \tag{30}$$

where

ω_n : angular natural frequency of the n'th mode

$$C_n = \frac{\Lambda_n \cdot \omega_n}{\sqrt{4\pi^2 + \Lambda_n^2}}$$

: damping coefficient of the n'th mode

Λ_n : logarithmic decrement of the n'th mode

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \tan^{-1}\left[\int F_{W2}(x)\sin\varepsilon_2(x) X_n(x) dx \Big/ \int F_{W2}(x)\cos\varepsilon_2(x) X_n(x) dx\right]$$

$$\delta_2 = \tan^{-1}\left[\left\{-2C_n\frac{(2\omega_e)}{\omega_n^2}\right\} \Big/ \left\{1 - \frac{(2\omega_e)^2}{\omega_n^2}\right\}\right]$$

The ship-hull has infinite number of vibratory modes, but, when $2\omega_e$ is equal to ω_1, the ratio of $|A_2(t)|/|A_1(t)|$ is about 0.01. Hence only the two-noded vibration is considered in what follows.

Two parameters λ_1 and ω_1 are obtained from the model experiments mentioned before, and the following mode of free-free beam is employed as the characteristic shape.

$$X_1(x) = C\left[(\cosh aL - \cos aL)\left\{\cosh a(x+\tfrac{L}{2}) + \cos a(x+\tfrac{L}{2})\right\} - (\sinh aL + \sin aL)\left\{\sinh a(x+\tfrac{L}{2}) + \sin a(x+\tfrac{L}{2})\right\}\right] \quad (31)$$

where $aL = 4.73$, and C is an arbitrary constant. Judging from the record of pressure in free vibration, this mode seems appropriate.

4.2 Dynamic Deflection, Longitudinal Bending Moment

Finally, the dynamic deflection of the two-noded vibration is given by

$$z(x,t) = A X_1(x) \sin(2\omega_e t + \delta) \quad (32)$$

The longitudinal bending moment at midship section is

$$M = \int_{-L/2}^{0} \{ F_i(x,t) + F_R(x,t) + F_{stat}(x) + F_g(x) + F_{WA}(x,t) \} x\,dx \quad (33)$$

where

$$F_i(x,t) = -\frac{w(x)}{g}\ddot{z}(x,t)$$

$$F_R(x,t) = -m_a(x)\ddot{z} - C_W(x)\dot{z} - 2\rho g\, b(x) z$$

m_a: added mass for heaving (calculated in 2-dim)
C_W: wave damping coefficient (")

$$F_{stat}(x) = \rho g S_W(x)$$

S_W: area of submerged section

$$F_g(x) = -w(x)$$

$$F_{WA}(x,t) = F_{W1}(x)\sin(\omega_e t + \varepsilon_1(x)) + F_{W2}(x)\sin(2\omega_e t + \varepsilon_2(x)) + \cdots\cdots$$

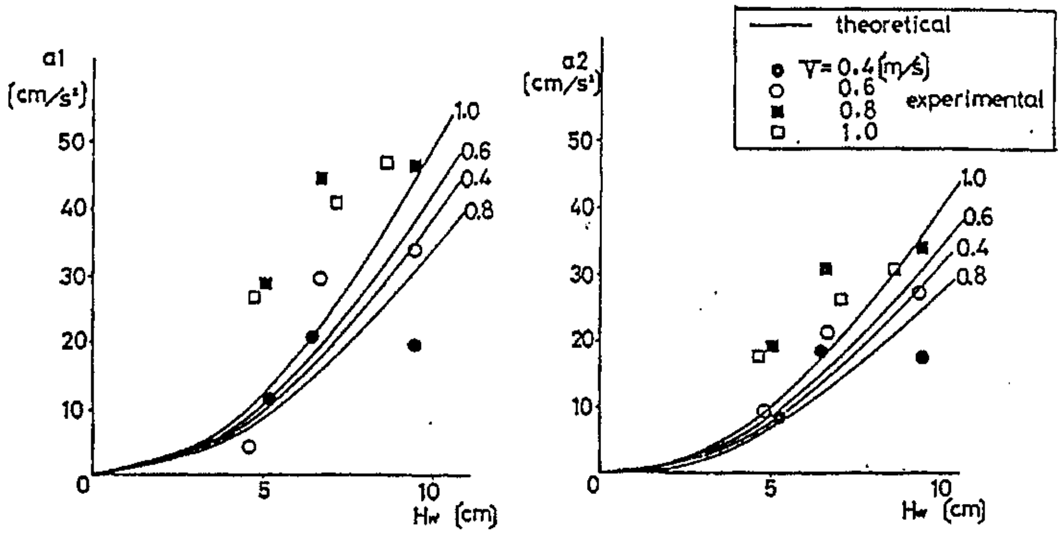
From the equation (33), the first order bending moment M_1 with encounter frequency ω_e and the second order bending moment M_2 with frequency $2\omega_e$ are derived as follows.

$$M_1 = \int_{-L/2}^{0} \{ F_{W1}(x)\sin(\omega_e t + \varepsilon_1(x)) \} x\,dx \quad (34)$$

$$M_2 = -\int_{-L/2}^{0} \left[\left\{ \frac{w(x)}{g} + m_a(x) \right\} \ddot{z} + C_W(x)\dot{z} + 2\rho g\, b(x) z - F_{W2}(x)\sin(2\omega_e t + \varepsilon_2(x)) \right] x\,dx \quad (35)$$

4.3 Comparison between Calculation and Experiment

Comparison between calculated value and experimental data of vertical acceleration and longitudinal bending moment in midship section is shown in Fig.9, and that of pressure in Fig.10.



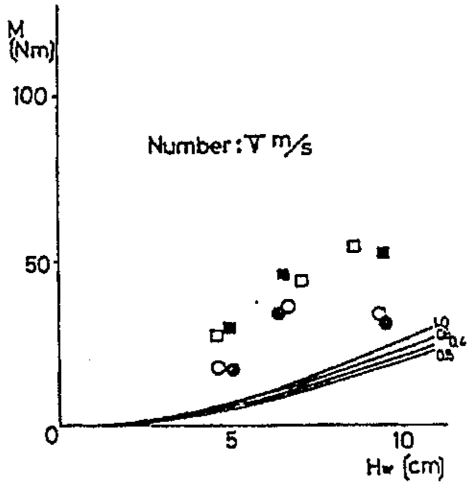


Fig.9 Oscillatory response of vertical acceleration and vertical bending moment ($f_e = f_1/2$)

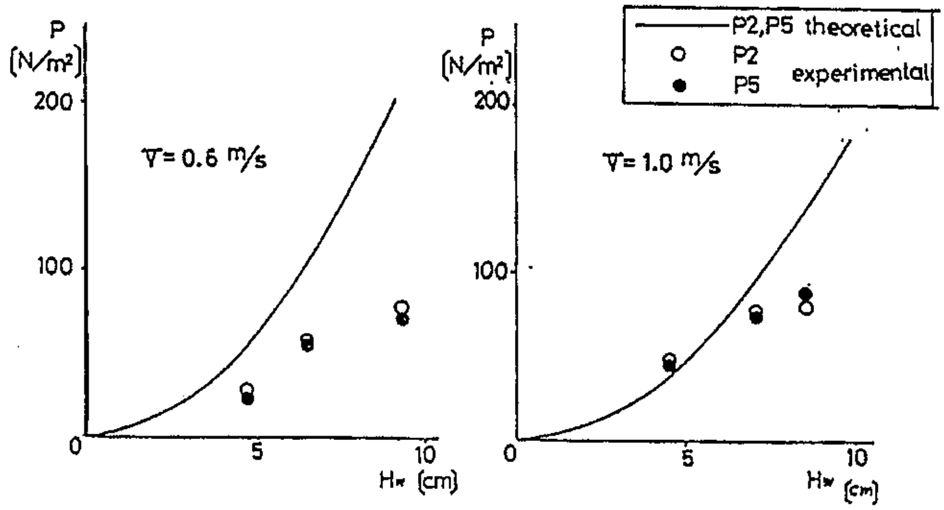


Fig.10 Oscillatory response of pressure at the bottom($f_e = f_1/2$)

Fig.11 shows comparison between theoretical values of vertical bending moment at midship by the first order exciting force, M_1, and that by the second order exciting force, M_2 . It is found that M_2 is comparable to M_1 .

The calculated value shows fairly good agreement with experimental data.

5. CONSIDERATION

The ship-hull response to waves can be represented by a simplified model like the one shown in Fig.12, and the region indicated with thick line is the object of this paper.

From the theoretical point of view, the calculation method employed in this paper is an approximate one; therefore, theoretical and experimental approaches on many examples such as ship forms and drafts are needed in order to confirm the validity of the calculation method used in this paper.

At present, there are few results of the fundamental study on nonlinear phenomena, so it is necessary to carry out the fundamental study further both theoretically and experimentally.[17]

As for ship design, the wave induced ship-hull vibration involving springing and whipping simultaneously is seriously important. Some prediction methods need to be set up.

6. CONCLUDING REMARKS

From the experimental and theoretical approaches on springing caused by higher-order exciting force, the following results have been obtained.

1) By the calculation method of the second order exciting force proposed in this paper, the experimental data on acceleration, longitudinal bending moment, and pressure can be explained well.
2) Judging from the experimental data of pressure, the difference of magnitude of the second order pressure between in fore part and in aft part is small. Hence, the reasoning that there exists a direct relation between the higher-order wave exciting force and the wave elevation along the hull surface seems inappropriate.
3) As for the ship model, it is possible to use one solid body made of elastic material in the experiment of two-noded ship-hull vibration, if the material is chosen carefully.
4) According to a trial calculation on the effect of increase in draft, the amplitude of vibratory responses decreases to the half owing to increase in draft by 50 percent.

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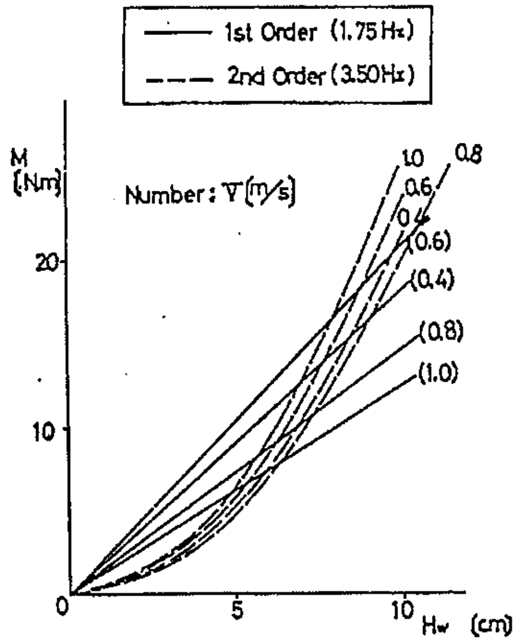

Fig.11 Comparison between theoretical values of vertical bending moment at midship induced by the fist-order and the second-order wave forces respectively ($f_e = f_1/2$)

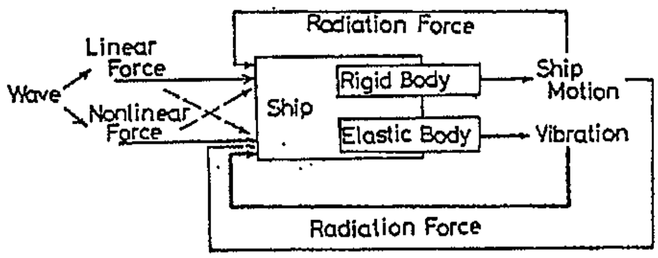

Fig.12 Simplified model of wave-induced vibration

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